1. **Natural numbers**: The numbers which are used for counting are called Natural numbers and represented with letter N

   N={1,2,3,4,5,……}

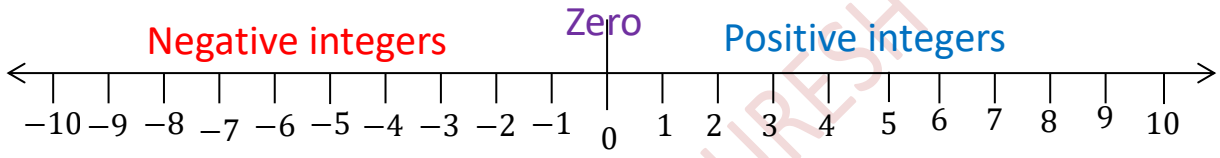
2. **Whole numbers**: If '0' is added to Natural numbers then they are called Whole numbers. And is denoted by 'W'

   W={0,1,2,3,4,5,……}

3. **Integers**: Combination of positive and negative numbers Including 0 are called Integers and represented by 'Z' or 'I'.

   Z={… … − 4, −3, −2, −1,0,1,2,3,4, … … … }

4. Integers number line



5. **Addition of integers**:

(i) When two positive integers are added, we get a positive integer.

e.g. (+5) + (+6) = +11

(ii) When two negative integers are added, we get a negative integer.

e.g. (−5) + (−6) = −11

(iii) When one positive and one negative integer are added we subtract them as whole numbers by considering the numbers without their sign and then put the sign of the bigger number with the subtraction obtained.

e.g. (+8) + (−5) = 3 , (−8) + (+5) = −3 , −7 + 5 = −2, 10 − 7 = 3,

−(+) = −
−(−) = +
+(−) = −

6. **Multiplication of integers**:

(i) If the signs of two integers are same then the product is positive integer.

e.g. (+3) × (+5) = 15, (−4) × (−3) = 12

(ii) If the signs of two integers are different then the product is negative integer.

e.g. (+3) × (−5) = −15, (−3) × (+5) = −15, (−4) × (+3) = −12, (+4) × (−3) = −12

7. **Division of integers**:

(i) If the signs are same then the quotient is positive.

e. g. 12 ÷ 3 = 4, (−12) ÷ (−3) = 4

(ii) If the signs are different then the quotient is negative.

e. g. (−12) ÷ 3 = −4, 12 ÷ (−3) = −4

8. Division by zero is not defined

$\frac{1}{0}, \frac{3}{0}, \frac{-51}{0}, \frac{-8}{0}$, are not defined

9. $0 \in W$ (0 belongs to whole numbers)
10. $0 \notin N$ (0 does not belong to natural numbers)
11. $-3 \in Z$ (-3 belongs to integers)

PROPERTIES OF ADDITION AND SUBTRACTION OF INTEGERS

1. Closure under Addition:

$(i)\ 17 + 23 = 40$

$(ii)(-10) + 3 = -7$

$(iii)(-75) + 18 = -57$

$(iv)\ 19 + (-25) = -6$

$(v)\ 27 + (-27) = 0$

$(vi)(-20) + 0 = -20$

$(vii)(-35) + (-10) = -45$

The sum of two integers always an integer. So, integers are closed under addition

For any two integers a and b, $(a + b)$ is an integer

2. **Closure under Subtraction:**

$(i)\ \ 7 - 9 = -2$

$(ii)\ \ 17 - (-21) = 17 + 21 = 39$

$(iii)\ (-8) - (-14) = -8 + 14 = 6$

$(iv)\ (-21) - (-10) = -21 + 10 = -11$

$(v)\ \ 32 - (-17) = 32 + 17 = 49$

$(vi)\ \ (-18) - (-18) = -18 + 18 = 0$

$(vii)\ (-29) - 0 = -29$

The subtraction of two integers always an integer. So, integers are closed under Subtraction.

If a and b are two integers then $(a - b)$ is also an integer.

3. **Commutative Property**:

$(i)\ 5 + (-6) = -1$ and $(-6) + 5 = -1$

(ii) $(-8) + (-9) = -17$ and $(-9) + (-8) = -17$

(iii) $(-23) + 32 = -11$ and $32 + (-23) = -11$

(iv) $-45) + 0 = -45$ and $0 + (-45) = -45$

The sum of two integers is not changed when the order is changed.

Addition is commutative for integers.

For any two integers a and b, we can say $\boldsymbol{a + b = b + a}$

$(i)\ 5 - (-3) = 5 + 3 = 8$ *and* $(-3) - 5 = -3 - 5 = -8$

$(ii)\ 10 - 5 = 5$ *and* $5 - 10 = -5$

The subtraction of two integers is changed when the order is changed.

Subtraction is not commutative for integers.

4. **Associative Property:**

$(i)(-5) + [(-3) + (-2)] = (-5) + (-5) = -10$ and $[(-5) + (-3)] + (-2) = (-8) + (-2) = -10.$

$(ii)(-3) + [1 + (-7)] = (-3) + (-6) = -9$ and $[(-3) + 1] + (-7) = -2 + (-7) = -9$

Addition is associative for integers.

For any integers a, b and c, we can say $\boldsymbol{a + (b + c) = (a + b) + c}$

5. **Additive Identity:** **Zero** is an additive identity for integers.

(i) $(-8) + 0 = -8$ | (iii) $(-23) + 0 = -23$

(ii) $0 + (-8) = -8$ | (iv) $0 + (-37) = -37$

For any integer a, $\boldsymbol{a + 0 = a = 0 + a}$

TRY THESE

1. **Write a pair of integers whose sum gives**

(a) a negative integer

Sol: $10, -15$ *and* $-12, 8$

$10 + (-15) = -5$; $-12 + 8 = -4$

(b) Zero

Sol: $-6, 6$ *and* $17, -17$

$(-6) + 6 = 0$ *and* $17 + (-17) = 0$

(c) An integer smaller than both the integers

Sol: $-5, -3$ *and* $-8, -10$

$(-5) + (-3) = -8$ *and* $(-8) + (-10) = -18$

(d) An integer smaller than only one of the integers.

Sol: $-6, 12$ *and* $15, -9$

$(-6) + 12 = 6$ *and* $17 + (-9) = 8$

(e) an integer greater than both the integers.

Sol: $5, 12$ *and* $15, 24$

$5 + 12 = 17$ *and* $15 + 24 = 39$

2. **Write a pair of integers whose difference gives**

(a) a negative integer

Sol: $5, 12$ *and* $-8, 4$

$5 - 12 = -7$ *and* $(-8) - 4 = -12$

(b) zero.

Sol: $14, 14$ *and* $-25, -25$

$14 - 14 = 0$ *and* $(-25) - (-25) = -25 + 25 = 0$

(c) an integer smaller than both the integers

Sol: $-4, 10$ *and* $8, 15$

$(-4) - 10 = -14$ *and* $8 - 15 = -7$

(d) an integer greater than only one of the integers.

Sol: $22,10\ and\ -11,-3$

$22-10=12\ and\ (-11)-(-3)=-11+3=-8$

(e) an integer greater than both the integers

Sol: $-5,-12\ \ and\ 6,-15$

$(-5)-(-12)=-5+12=7\ and\ 6-(-15)=6+15=21$

EXAMPLE 1: Write down a pair of integers whose

(a) sum is –3

Sol: (-1)+(-2)=-3 or (-8)+5=-3 or 10+(-7)=3

(b) difference is –5

Sol: (–9) – (– 4) = –5 or (–2) – 3 = –5 or 10-15=-5

(c) difference is 2

Sol: (–7) – (–9) = 2 or 1 – (–1) = 2 or 0-(-2)=2 or 10-8=2

(d) sum is 0

Sol: (–10) + 10 = 0 or 5 + (–5) = 0

EXERCISE 1.1

1. Write down a pair of integers whose:

(a) sum is –7

Sol: $(-5)+(-2)=-7\ or\ (-10)+3=-7\ or\ 15+(-22)=-7$

(b) difference is –10

Sol: $(-5)-(5)=-10\ or\ 20-10=10\ or\ -30-(-20)=-10$

(c) sum is 0

Sol: $5+(-5)=0\ \ or\ (-9)+9=0\ or\ \ 15+(-15)=0$

2. (a) Write a pair of negative integers whose difference gives 8.

Sol: $(-5)-(-13)=-5+13=8\ \ or\ \ (-10)-(-18)=-10+18=8$

(b) Write a negative integer and a positive integer whose sum is –5.

Sol: $-10+5=-5\ \ or\ \ -12+7=-5\ \ or\ \ -20+15=-5$

(c) Write a negative integer and a positive integer whose difference is –3.

Sol: $(-1)-2=-1-2=-3\ \ or\ (-2)-1=-2-1=-3$

3. In a quiz, team A scored – 40, 10, 0 and team B scored 10, 0, – 40 in three successive rounds. Which team scored more? Can we say that we can add integers in any order.

Sol: Total score of team A= $-40+10+0=-30$

Total score of team B= $10+0+(-40)=10-40=-30$

Scores of both the teams are same

Yes, we can add integers in any order.

4. Fill in the blanks to make the following statements true:

$(i)\quad (-5) + (-8) = (-8) + (-5)$

$(ii)\quad -53 + 0 = -53$

$(iii)\quad 17 + (-17) = 0$

$(iv)\quad [13 + (-12)] + (-7) = 13 + [(-12) + (-7)]$

$(v)\quad (-4) + [15 + (-3)] = [-4 + 15] + (-3)$

MULTIPLICATION OF INTEGERS

1. Multiplication of a Positive and a Negative Integer:

$3 \times (-5) = (-5) + (-5) + (-5) = -15$

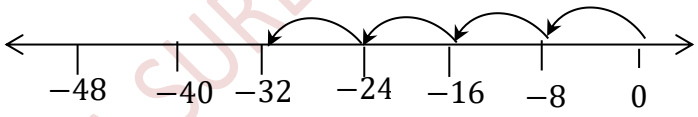
$5 \times (-4) = (-4) + (-4) + (-4) + (-4) + (-4) = -20$

(positive integer) × (negative integer)= (negative integer) × (positive integer)= negative integer

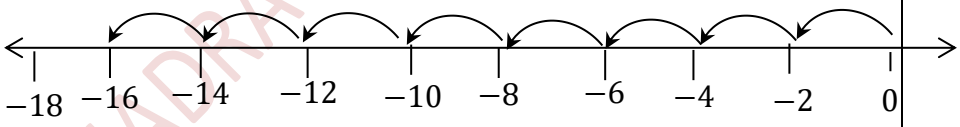
TRY THESE

Find using number line

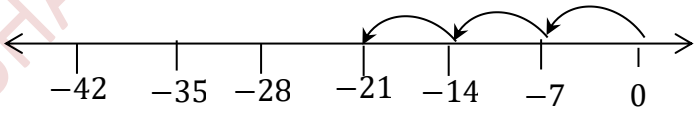
$(i)\ 4 \times (-8) = -32$



$(ii)\ 8 \times (-2) = -16$



$(iii)\ 3 \times (-7) = -21$



$(i)\ 6 \times (-19) = -114$ $(ii)\ 12 \times (-32) = -384$ $(iii)\ 7 \times (-22) = -154$

For any two positive integers a and b we can say $\boldsymbol{a \times (-b) = (-a) \times b = -(a \times b)}$

TRY THESE

1. Find

$(\boldsymbol{a})\ \mathbf{15} \times (-\mathbf{16}) = -240$ $(\boldsymbol{b})\ \mathbf{21} \times (-\mathbf{32}) = -672$

$(\boldsymbol{c})\ (-\mathbf{42}) \times \mathbf{12} = -504$ $(\boldsymbol{d})\ -\mathbf{55} \times \mathbf{15} = -825$

2. Check if (a) 25 × (–21) = (–25) × 21

Sol: LHS $= 25 \times (-21) = -(25 \times 21) = -525$;

RHS $= (-25) \times 21 = -(25 \times 21) = -525$

LHS=RHS; Hence verified

(b) (–23) × 20 = 23 × (–20)

Sol: $\text{LHS} = (-23) \times 20 = -(23 \times 20) = -460$

$\text{RHS} = 23 \times (-20) = -(23 \times 20) = -460$

LHS=RHS ; Hence verified

Examples: $(i)\ 26 \times (-35) = (-26) \times 35$ $(ii)\ (-12) \times 28 = 12 \times (-28)$

Multiplication of two Negative Integers:

TRY THESE

(i) Starting from (–5) × 4, find (–5) × (– 6)

$Sol: (-5) \times 4 = -20$	$(-5) \times 0 = 0$	$(-5) \times (-4) = 20$
$(-5) \times 3 = -15$	$(-5) \times (-1) = 5$	$(-5) \times (-5) = 25$
$(-5) \times 2 = -10$	$(-5) \times (-2) = 10$	$(-5) \times (-6) = 30$
$(-5) \times 1 = -5$	$(-5) \times (-3) = 15$	$\therefore (-5) \times (-6) = 30$

(ii) Starting from (– 6) × 3, find (– 6) × (–7)

$Sol: (-6) \times 3 = -18$	$(-6) \times (-1) = 6$	$(-6) \times (-5) = 30$
$(-6) \times 2 = -12$	$(-6) \times (-2) = 12$	$(-6) \times (-6) = 36$
$(-6) \times 1 = -6$	$(-6) \times (-3) = 18$	$(-6) \times (-7) = 42$
$(-6) \times 0 = 0$	$(-6) \times (-4) = 24$	$\therefore (-6) \times (-7) = 42$

The product of two negative integers is a positive integer.

We multiply the two negative integers as whole numbers and put the positive sign before the product

For any two positive integers a and b, $(-\boldsymbol{a}) \times (-\boldsymbol{b}) = \boldsymbol{a} \times \boldsymbol{b}$

TRY THESE

Find: $(\boldsymbol{i})(-\mathbf{31}) \times (-\mathbf{100}) = 3100,$

$(\boldsymbol{ii})\ (-\mathbf{25}) \times (-\mathbf{72}) = 1800,$ $(\boldsymbol{iii})\ (-\mathbf{83}) \times (-\mathbf{28}) = 2384$

PROPERTIES OF MULTIPLICATION OF INTEGERS

1. Closure under Multiplication:

$(-20) \times (-5) = 100$	$(-30) \times 12 = -360$	$(-14) \times (-13) = 182$
$(-15) \times 17 = -255$	$(-15) \times (-23) = 345$	$12 \times (-30) = -360$

The product of two integers is again an integer. So we can say that integers are closed under multiplication.

For all integers a and b, a × b is an integer,

2. **Commutativity of Multiplication**

Statement 1	Statement 2	Inference
$3 \times (-4) = -12$	$(-4) \times 3 = -12$	$3 \times (-4) = (-4) \times 3$
$(-30) \times 12 = -360$	$12(-30) = -360$	$(-30) \times 12 = 12(-30)$
$(-15) \times (-10) = 150$	$(-10) \times (-15) = 150$	$(-15) \times (-10) = (-10) \times (-15)$
$(-35) \times (-12) = 420$	$(-12) \times (-35) = 420$	$(-35) \times (-12) = (-12) \times (-35)$
$(-17) \times 0 = 0$	$0 \times (-17) = 0$	$(-17) \times 0 = 0 \times (-17)$
$(-15) \times (-1) = 15$	$(-1) \times (-15) = 15$	$(-15) \times (-1) = (-1) \times (-15)$

Multiplication is commutative for integers.

For any two integers *a and b* : $\boldsymbol{a \times b = b \times a}$

3. **Multiplication by Zero**

For any integer *a* we have $\boldsymbol{a \times 0 = 0 \times a = 0}$

4. **Multiplicative Identity** :1 is the multiplicative identity for integers

$(-3) \times 1 = -3$	$1 \times 8 = 8$	$1 \times (-6) = -6$
$1 \times 5 = 5$	$1 \times (-5) = -5$	$7 \times 1 = 7$
$(-4) \times 1 = -4$	$3 \times 1 = 3$	

For any integer a we have, $\boldsymbol{a \times 1 = 1 \times a = a}$

Additive inverse:

We get additive inverse of an integer a when we multiply (–1) to a, i.e. $a \times (-1) = (-1) \times a = -a$

5. **Associativity for Multiplication:**

$[(7) \times (-6)] \times 4 = (-42) \times 4 = -168$ *and* $7 \times [(-6) \times 4] = 7 \times (-24) = -168$

$[(7) \times (-6)] \times 4 = 7 \times [(-6) \times 4]$

For any three integers *a, b and c we have* $\boldsymbol{(a \times b) \times c = a \times (b \times c)}$

6. **Distributive Property:**

For any integers *a, b and c*,

$\boldsymbol{(i) a \times (b + c) = a \times b + a \times c}$ is called distributivity of multiplication over addition

$\boldsymbol{(ii) a \times (b - c) = a \times b - a \times c}$ is called distributivity of multiplication over subtraction.

TRY THESE

(*i*) Is $10 \times [(6 + (-2)] = 10 \times 6 + 10 \times (-2)$?

Sol: $LHS = 10 \times [(6 + (-2)] = 10 \times 4 = 40$

$RHS = 10 \times 6 + 10 \times (-2) = 60 - 20 = 40$

LHS=RHS, So, given statement is true.

(ii) Is (–15) × [(–7) + (–1)] = (–15) × (–7) + (–15) × (–1)?

Sol: $LHS = (-15) \times [(-7) + (-1)] = (-15) \times (-8) = 120$

$RHS = (-15) \times (-7) + (-15) \times (-1) = 105 + 15 = 120$

LHS=RHS . So, given statement is true.

TRY THESE

(*i*) Is $10 \times (6 - (-2)] = 10 \times 6 - 10 \times (-2)$?

Sol: $\text{LHS} = 10 \times (6 - (-2)] = 10 \times (6 + 2) = 10 \times 8 = 80$

$\text{RHS} = 10 \times 6 - 10 \times (-2) = 60 + 20 = 80$

LHS=RHS . So, given statement is true

(ii) Is $(-15) \times [(-7) - (-1)] = (-15) \times (-7) - (-15) \times (-1)$?

Sol: $\text{LHS} = (-15) \times [(-7) - (-1)] = (-15) \times (-7 + 1) = (-15) \times (-6) = 90$

$\text{RHS} = (-15) \times (-7) - (-15) \times (-1) = 105 - 15 = 90$

LHS=RHS . So, given statement is true.

EXERCISE 1.2

1. Find each of the following products:

(*a*) $3 \times (-1) = -3$
(*b*) $(-1) \times 225 = -225$
(*c*) $(-21) \times (-30) = 630$
(*d*) $(-316) \times (-1) = 316$
(*e*) $(-15) \times 0 \times (-18) = 0$
(f) $(-12) \times (-11) \times (10) = 1320$
(g) $9 \times (-3) \times (-6) = 162$
(h) $(-18) \times (-5) \times (-4) = -360$
(i) $(-1) \times (-2) \times (-3) \times 4 = -24$
(j) $(-3) \times (-6) \times (-2) \times (-1) = 36$

2. Verify the following:

(a) $18 \times [7 + (-3)] = [18 \times 7] + [18 \times (-3)]$

Sol: $\text{LHS} = 18 \times [7 + (-3)] = 18 \times 4 = 72$

$\text{RHS} = [18 \times 7] + [18 \times (-3)] = 126 + (-54) = 72$

LHS=RHS. So, given statement is true

(b) $(-21) \times [(-4) + (-6)] = [(-21) \times (-4)] + [(-21) \times (-6)]$

Sol: $\text{LHS} = (-21) \times [(-4) + (-6)] = (-21) \times (-10) = 210$

$\text{RHS} = [(-21) \times (-4)] + [(-21) \times (-6)] = 84 + 126 = 210$

LHS=RHS. So, given statement is true

3. (i) For any integer a, what is $(-1) \times a$ equal to?

Sol: For any integer a, $(-1) \times a = -a$

(ii) Determine the integer whose product with (–1) is

(a) –22 (b) 37 (c) 0

Sol: $(a)\ (-1) \times (-22) = 22 \quad (b)\ (-1) \times 37 = -37 \quad (c)\ (-1) \times 0 = 0$

4. Starting from (–1) × 5, write various products showing some pattern to show (–1) × (–1) = 1

Sol: $(-1) \times 5 = -5$

$(-1) \times 4 = -4$

$(-1) \times 3 = -3$

$(-1) \times 2 = -2$

$(-1) \times 1 = -1$

$(-1) \times 0 = 0$

$(-1) \times (-1) = 1$

DIVISION OF INTEGERS

Multiplication Statement	Corresponding Division Statements	
$2 \times (-6) = (-12)$	$(-12) \div (-6) = 2$	$(-12) \div 2 = (-6)$
$(-4) \times 5 = (-20)$	$(-20) \div 5 = -4$	$(-20) \div (-4) = 5$
$(-8) \times (-9) = 72$	$72 \div (-9) = (-8)$	$72 \div (-8) = (-9)$
$(-3) \times (-7) = 21$	$21 \div (-7) = (-3)$	$21 \div (-3) = (-7)$
$(-8) \times 4 = (-32)$	$(-32) \div 4 = -8$	$(-32) \div (-8) = 4$
$5 \times (-9) = (-45)$	$(-45) \div (-9) = 5$	$(-45) \div 5 = (-9)$
$(-10) \times (-5) = 50$	$50 \div (-5) = (-10)$	$50 \div (-10) = (-5)$

When we divide a negative integer by a positive integer, we divide them as whole numbers and then put a minus sign (–) before the quotient.

$(+) \div (-) = -$
$(-) \div (+) = -$

For any two positive integers a and b $a \div (-b) = (-a) \div b = -(a \div b)$ where $b \neq 0$

When we divide a negative integer by a negative integer, we first divide them as whole numbers and then put a positive sign (+).

$(+) \div (+) = +$
$(-) \div (-) = +$

For any two positive integers a and b $(-a) \div (-b) = a \div b$ where $b \neq 0$

TRY THESE

Find: $(a)\ 125 \div (-25) = -5 \quad (b)\ 80 \div (-5) = -16 \quad (c)\ 64 \div (-16) = -4$

Find: **(a) (–36) ÷ (– 4)** =9 **(b) (–201) ÷ (–3)**=67 **(c) (–325) ÷ (–13)**=25

PROPERTIES OF DIVISION OF INTEGERS

1. Integers are not closed under division.

   $(-8) \div (-4) = 2$ is an integer and $(-4) \div (-8) = \frac{-4}{-8} = \frac{1}{2}$ is not an integer

   For any two integers a and b ; $a \div b$ **need not be an integer**.
2. Division is not commutative for integers.

   $(-8) \div (-4) \neq (-4) \div (-8)$

   For any two different integers a and b ; $a \div b \neq b \div a$
3. **Divided by zero :** For any integer a, $a \div 0$ is not defined.

$\frac{1}{0}, \frac{-5}{0}, \frac{3}{0}, \ldots$ *are not defined.*

4. **Divided by '1':**

$(-8) \div 1 = (-8)$	$(-13) \div 1 = -13$	$(-37) \div 1 = -37$
$(-11) \div 1 = -11$	$(-25) \div 1 = -25$	$(-48) \div 1 = -48$

Any integer divided by 1 gives the same integer.**For any integer a, $a \div 1 = a$**

5. If any integer is divided by (–1) it does not give the same integer.
6. Division is not associative for integers.
   $(-16) \div [4 \div (-2)] = (-16) \div 2 = -8$ and $[(-16) \div 4] \div (-2) = (-4) \div (-2) = 2$
   $[(-16) \div 4] \div (-2) \neq (-16) \div [4 \div (-2)]$

Exp 2 :In a test (+5) marks are given for every correct answer and (–2) marks are given for every incorrect answer. (i) Radhika answered all the questions and scored 30 marks though she got 10 correct answers. (ii) Jay also answered all the questions and scored (–12) marks though he got 4 correct answers. How many incorrect answers had they attempted?

Sol: (i) Marks for one correct answer=5

Marks for 10 correct answers=10×5=50

Radhika's score = 30

Marks obtained for incorrect answers = 30 – 50 = – 20

Marks for one incorrect answer = (–2)

Therefore, number of incorrect answers = (–20) ÷ (–2) = 10

(ii) Marks given for 4 correct answers = 5 × 4 = 20

Jay's score = –12

Marks obtained for incorrect answers = –12 – 20 = – 32

Marks given for one incorrect answer = (–2)

Therefore number of incorrect answers = (–32) ÷ (–2) = 16

EXAMPLE 3 A shopkeeper earns a profit of ₹ 1 by selling one pen and incurs a loss of 40 paise per pencil while selling pencils of her old stock. (i) In a particular month she incurs a loss of ₹ 5. In this period, she sold 45 pens. How many pencils did she sell in this period? (ii) In the next month she earns neither profit nor loss. If she sold 70 pens, how many pencils did she sell?

Sol: Profit earned by selling one pen = ₹ 1

Profit earned by selling 45 pens = ₹ 45, which we denote by + ₹ 45

Total loss given = ₹ 5, which we denote by – ₹ 5

Profit earned + Loss incurred = Total loss

Therefore, Loss incurred = Total Loss – Profit earned = ₹ (– 5 – 45) = ₹ (–50) = –5000 paise

Loss incurred by selling one pencil = 40 paise which we write as – 40 paise So, number of pencils sold = (–5000) ÷ (– 40) = 125.

(ii) In the next month there is neither profit nor loss.

So, Profit earned + Loss incurred = 0 i.e., Profit earned = – Loss incurred

Now, profit earned by selling 70 pens = ₹ 70 Hence, loss incurred by selling pencils = ₹ 70 which we indicate by – ₹ 70 or – 7,000 paise. Total number of pencils sold = (–7000) ÷ (– 40) = 175 pencils.

EXERCISE 1.3

1. Evaluate each of the following:

$(a)\ (-30) \div 10 = -3$

$(b)\ 50 \div (-5) = -10$

$(c)\ (-36) \div (-9) = 4$

$(d)\ (-49) \div (49) = -1$

$(e)\ 13 \div [(-2) + 1] = 13 \div (-1) = -13$

$(f)\ 0 \div (-12) = 0$

$(g)\ (-31) \div [(-30) + (-1)]$
$= (-31) \div (-31) = 1$

$(h)\ [(-36) \div 12] \div 3$
$= (-3) \div 3 = -1$

$(i)\ [(-6) + 5)] \div [(-2) + 1]$
$= (-1) \div (-1) = 1$

2. Verify that $a \div (b + c) \neq (a \div b) + (a \div c)$ for each of the following values of a, b and c

$(a)\ a = 12, b = -4, c = 2$

Sol: $a \div (b + c) = 12 \div (-4 + 2) = 12 \div (-2) = 6$

$(a \div b) + (a \div c) = (12 \div -4) + (12 \div 2) = 3 + 6 = 9$

$a \div (b + c) \neq (a \div b) + (a \div c)$

$(b)\ a = (-10), b = 1, c = 1$

Sol: $a \div (b + c) = (-10) \div (1 + 1) = (-10) \div (2) = -5$

$(a \div b) + (a \div c) = (-10 \div -1) + (-10 \div 1) = (-10) + (-10) = -20$

$a \div (b + c) \neq (a \div b) + (a \div c)$

3. Fill in the blanks.

$(a)\ 369 \div 1 = 369$

$(b)\ (-75) \div 75 = -1$

$(c)\ (-206) \div (-206) = 1$

$(d)\ (-87) \div (-1) = 87$

$(e)\ (-87) \div 1 = -87$

$(f)\ (-48) \div 48 = -1$

$(g)\ 20 \div (-10) = -2$

$(h)\ (-12) \div (4) = -3$

4. Write five pairs of integers (a, b) such that a ÷ b = –3. One such pair is (6, –2) because 6 ÷ (–2) = (–3).

Sol:

(i) $(9,-3)$ because $9 \div (-3) = -3$

(ii) $(-12,4)$ because $(-12) \div 4 = -3$

(iii) $(-15,5)$ because $(-15) \div 5 = -3$

(vi) $(30,-10)$ because $30 \div (-10) = -3$

(v) $(45,-15)$ because $45 \div (-15) = -3$

5. The temperature at 12 noon was 10°C above zero. If it decreases at the rate of 2°C per hour until midnight, at what time would the temperature be 8°C below zero? What would be the temperature at mid-night?

Sol: The temperature at 12 noon = 10°C above zero=+10°C

Decrease rate per 1 hour is 2°C=-2°C

Time taken for 8°C below zero= Time taken to decrease $\frac{10°C-(-8°C)}{2°C} = \frac{18°C}{2°C} = 9\ hours$

The temperature be 8°C below zero at 9PM.

(ii) The temperature at mid-night= 10°C+ Temperature decreased in 12 hours

$= 10°C + (-2°C \times 12) = 10°C + (-24°C) = -14°C$

The temperature at mid-night will be 14°C below zero

6. In a class test (+ 3) marks are given for every correct answer and (–2) marks are given for every incorrect answer and no marks for not attempting any question. (i) Radhika scored 20 marks. If she has got 12 correct answers, how many questions has she attempted incorrectly? (ii) Mohini scores –5 marks in this test, though she has got 7 correct answers. How many questions has she attempted incorrectly?

Sol: Marks for 1 correct answer $= +3$, Marks for 1 incorrect answer $= -2$

Marks for not attempting=0

(i) Marks for 12 correct answers=12×3=36

Radhika scored 20 marks

Marks for incorrect answers $= 20 - 36 = -16$

Number of incorrect answers $= (-16) \div (-2) = 8$

(ii) Marks for 7 correct answers=7×3=21

Mohini scores $= -5$

Marks for incorrect answers $= -5 - 21 = -26$

Number of incorrect answers $= (-26) \div (-2) = 13$

Mohini attempted 13 questions incorrectly.

7. An elevator descends into a mine shaft at the rate of 6 m/min. If the descent starts from 10 m above the ground level, how long will it take to reach – 350 m.

Sol: Time taken to elevator descends of 6 m= 1 min

Total distance covered by elevator $= 10 - (-350) = 10 + 350 = 360$ m

Time taken to elevator descends $360\text{m} = 360 \div 6 = 60$ min $= 1$ hour

CHAPTER 2

VII-MATHEMATICS-NCERT

2. Fractions and Decimals (notes)

PREPARED BY: BALABHADRA SURESH

1. A fraction is a number representing a part of a whole. The whole may be a single object or a group of objects
2. $\frac{5}{12}$ is a fraction . Here 5 is called the numerator and 12 is called the denominator
3. **Proper Fraction**: In a fraction , the numerator is less than the denominator is called proper fraction.
   $Ex: \frac{2}{5}, \frac{3}{11}, \frac{5}{21}, \ldots$
4. **Improper Fraction**: The fractions, where the numerator is bigger than the denominator are called improper fractions.
   $Ex: \frac{7}{5}, \frac{13}{11}, \frac{35}{21}, \ldots$
5. **Mixed Fractions**: Mixed fraction has a combination of a whole and a part.
6. The mixed fraction will be written as **Quotient** $\frac{\text{Remainder}}{\text{Divisor}}$
   $Ex: 3\frac{2}{5}, 1\frac{3}{11}, 2\frac{5}{21}, \ldots$
7. we can express a mixed fraction as an improper fraction as $\frac{(\text{Whole}\times\text{Denominator})+\text{Numerator}}{\text{Denominator}}$
   $Ex: 3\frac{2}{5} = \frac{17}{5}, \quad 1\frac{3}{11} = \frac{14}{11}, \quad 2\frac{5}{21} = \frac{47}{21}, \ldots$
8. If we multiply (or divide)the numerator and the denominator of the given fraction with same number we get an equivalent fraction.
   $Ex: (i)\ \frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{10}{15} = \cdots$
   $(ii)\ \frac{3}{7} = \frac{6}{14} = \frac{9}{21} = \frac{15}{35} = \cdots$
9. A fraction is said to be in the simplest (or lowest) form if its numerator and denominator have no common factor except 1.
10. **Like fractions**: Fractions with same denominators are called like fractions.
    $Ex: (i)\ \frac{3}{5}, \frac{1}{5}, \frac{11}{5}, ..$
11. **Unlike fractions**: Fractions with different denominators are called unlike fractions.
    $Ex: \frac{3}{5}, \frac{5}{7}, \frac{2}{11}, \ldots$
12. For two fractions with the same denominator, the fraction with the greater numerator is greater
    $Ex: \frac{7}{5} > \frac{4}{5}, \quad \frac{11}{23} > \frac{10}{23}$
13. If the numerator is the same in two fractions, the fraction with the smaller denominator is greater of the two.
    $Ex: \frac{7}{15} > \frac{7}{27}, \quad \frac{11}{23} > \frac{11}{29}$
14. Addition and subtraction of like fractions:
    $\frac{x}{a} + \frac{y}{a} = \frac{x+y}{a} \quad and \quad \frac{x}{a} - \frac{y}{a} = \frac{x-y}{a}$

$Ex: (i)\ \frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}=\frac{5}{7}$

$(ii)\ \frac{23}{35}-\frac{9}{35}=\frac{23-9}{35}=\frac{14}{25}$

$(iii) 5\frac{2}{9}+2\frac{5}{9}=\frac{47}{9}+\frac{23}{9}=\frac{47+23}{9}=\frac{70}{9}=7\frac{7}{9}$

15. Addition and subtraction of unlike fractions

$\frac{x}{a}+\frac{y}{b}=\frac{x\times b+y\times a}{a\times b}$ *and* $\frac{x}{a}-\frac{y}{b}=\frac{x\times b-y\times a}{a\times b}$

$Ex: \frac{2}{3}+\frac{1}{4}=\frac{2\times 4+1\times 3}{3\times 4}=\frac{8+3}{12}=\frac{11}{12}$

Multiplication of a Fraction by a Whole Number:

To multiply a whole number with a proper or an improper fraction, we multiply the whole number with the numerator of the fraction, keeping the denominator same.

$Ex: (i)\ 2\times\frac{5}{3}=\frac{2\times 5}{3}=\frac{10}{3}\ \ (ii)\ \frac{7}{5}\times 4=\frac{7\times 4}{5}=\frac{28}{5}$

Try These

1. Find the product. If the product is an improper fraction express it as a mixed fraction.

$(a)\frac{2}{7}\times 3=\frac{2\times 3}{7}=\frac{6}{7}$

$(b)\frac{9}{7}\times 6=\frac{9\times 6}{7}=\frac{54}{7}=7\frac{5}{7}$

$(c)\ 3\times\frac{1}{8}=\frac{3\times 1}{8}=\frac{3}{8}$

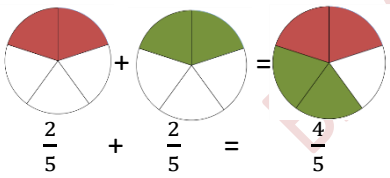
$(d)\ \frac{13}{11}\times 6=\frac{13\times 6}{11}=\frac{78}{11}=7\frac{1}{11}$

$7\overline{)54}$ quotient 7, $(-)49$, remainder 5

$11\overline{)78}$ quotient 7, $(-)77$, remainder 1

2. Represent pictorially : $2\times\frac{2}{5}=\frac{4}{5}$

Sol:



Multiplication of a Mixed Fraction by a Whole Number:

To multiply a mixed fraction to a whole number, first convert the mixed fraction to an improper fraction and then multiply.

Ex: $(i)\ 3\times 2\frac{5}{7}=3\times\frac{19}{7}=\frac{3\times 19}{7}=\frac{57}{7}=8\frac{1}{7}$

$(ii)\ 2\times 4\frac{2}{5}=2\times\frac{22}{5}=\frac{44}{5}=8\frac{4}{5}$

Try These

$(i) 5\times 2\frac{3}{7}=5\times\frac{17}{7}=\frac{5\times 17}{7}=\frac{85}{7}=12\frac{1}{7}$

$(ii)\ 1\frac{4}{9} \times 6 = \frac{13}{9} \times 6 = \frac{13 \times 6}{9} = \frac{78}{9} = 8\frac{6}{9} = 8\frac{2}{3}$

Fraction as an operator 'of' : 'of' represents multiplication

Ex: $\frac{1}{2}\ of\ 3 = \frac{1}{2} \times 3 = \frac{3}{2} = 1\frac{1}{2}$

Try These

$(i)\ \ \frac{1}{2} of\ 10 = \frac{1}{2} \times 10 = \frac{1 \times 10}{2} = \frac{10}{2} = 5$

$(ii)\ \ \frac{1}{4} of\ 16 = \frac{1}{4} \times 16 = \frac{1 \times 16}{4} = \frac{16}{4} = 4$

$(iii)\ \ \frac{2}{5} of\ 25 = \frac{2}{5} \times 125 = \frac{2 \times 25}{5} = \frac{50}{5} = 10$

Exp 1: In a class of 40 students $\frac{1}{5}$ of the total number of students like to study English, $\frac{2}{5}$ of the total number like to study Mathematics and the remaining students like to study Science. (i) How many students like to study English? (ii) How many students like to study Mathematics? (iii) What fraction of the total number of students likes to study Science?

Sol: Total number of students = 40.

(i)Number of students like to study English $= \frac{1}{5}\ of\ 40 = \frac{1}{5} \times 40 = \frac{40}{5} = 8$

(ii)Number of students like to study Mathematics $= \frac{2}{5}\ of\ 40 = \frac{2}{5} \times 40 = \frac{80}{5} = 16$

(iii) The number of students who like English and Mathematics = 8 + 16 = 24

The number of students who like Science = 40 – 24 = 16.

Thus, the required fraction is $\frac{16}{40} = \frac{2}{5}$

Exercise 2.1

1. Which of the drawings (a) to (d) show :

(i) $2\times\frac{1}{5}$ (ii) $2\times\frac{1}{2}$ (iii) $3\times\frac{2}{3}$ (iv) $3\times\frac{1}{4}$

(a) 

(b) 

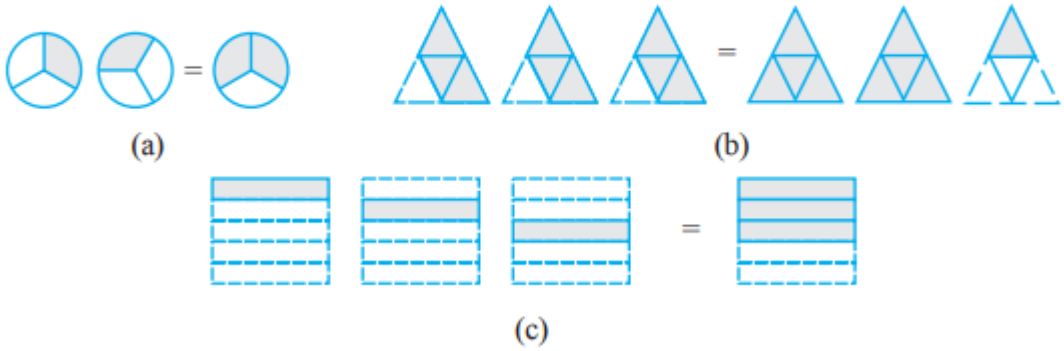
(c) 

(d) 

Sol: $(i) - (d)$ $(ii) - (b)$ $(iii) - (a)$ $(iv) - (c)$

2. **Some pictures (a) to (c) are given below. Tell which of them show:**

(i) $3\times\frac{1}{5}=\frac{3}{5}$ (ii) $2\times\frac{1}{3}=\frac{2}{3}$ (iii) $3\times\frac{3}{4}=2\frac{1}{4}$



(a) (b) (c)

Sol: (i) – (c) , (ii) – (a) , (iii) – (b)

3. **Multiply and reduce to lowest form and convert into a mixed fraction:**

Sol:

$(i)\ 7\times\frac{3}{5}=\frac{21}{5}=4\frac{1}{5}$

$(ii)\ 4\times\frac{1}{3}=\frac{4}{3}=1\frac{1}{3}$

$(iii)\ 2\times\frac{6}{7}=\frac{12}{7}=1\frac{5}{7}$

$(iv)\ 5\times\frac{2}{9}=\frac{10}{9}=1\frac{1}{9}$

$(v)\ \frac{2}{3}\times 4=\frac{8}{5}=1\frac{3}{5}$

$(vi)\ \frac{5}{2}\times 6=\frac{30}{2}=15$

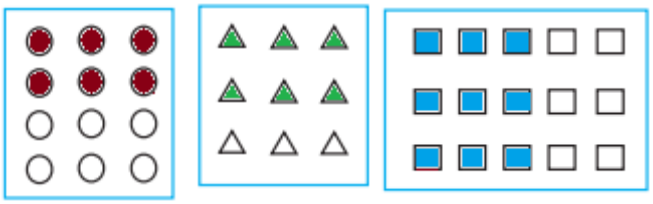
$(vii)\ 11\times\frac{4}{7}=\frac{44}{7}=6\frac{2}{7}$

$(viii)\ 20\times\frac{4}{5}=4\times 4=16$

$(ix)\ 13\times\frac{1}{3}=\frac{13}{3}=4\frac{1}{3}$

$(x)\ 15\times\frac{3}{5}=3\times 3=9$

4. **Shade: (i) $\frac{1}{2}$ of the circles in box (a) (ii) $\frac{2}{3}$ of the triangles in box (b) (iii) $\frac{3}{5}$ of the squares in box (**



(a) (b) (c)

5. **Find:**

$(i)\frac{1}{2}\ of\ 24=\frac{1}{2}\times 24=\frac{24}{2}=12$

$(ii)\frac{1}{2}\ of\ 46=\frac{1}{2}\times 46=\frac{46}{2}=23$

$(i)\frac{2}{3}\ of\ 18=\frac{2}{3}\times 18=\frac{2\times \cancel{18}^{6}}{\cancel{3}}=2\times 6=12$

6. Multiply and express as a mixed fraction :

$(a)\quad 3\times 5\frac{1}{5}=3\times\frac{26}{5}=\frac{3\times 26}{5}=\frac{78}{5}=15\frac{3}{5}$

$(b)\quad 5\times 6\frac{3}{4}=5\times\frac{27}{4}=\frac{5\times 27}{4}=\frac{135}{4}=33\frac{3}{5}$

$(c)\quad 7\times 2\frac{1}{4}=7\times\frac{9}{4}=\frac{7\times 9}{4}=\frac{63}{4}=15\frac{3}{4}$

$(d)\quad 4\times 6\frac{1}{3}=4\times\frac{19}{3}=\frac{4\times 19}{3}=\frac{76}{3}=25\frac{1}{3}$

$(e)\quad 3\frac{1}{4}\times 6=\frac{13}{4}\times 6=\frac{13\times \cancel{6}^{3}}{\cancel{4}_{2}}=\frac{39}{2}=19\frac{1}{2}$

$(f)\quad 3\frac{2}{5}\times 8=\frac{17}{5}\times 8=\frac{17\times 8}{5}=\frac{136}{5}=27\frac{1}{5}$

7. Find:

$(a)\ (i)\ \frac{1}{2}of\ 2\frac{3}{4}=\frac{1}{2}\times\frac{11}{4}=\frac{1\times 11}{2\times 4}=\frac{11}{8}=1\frac{3}{8}$

$(ii)\ \frac{1}{2}of\ 4\frac{2}{9}=\frac{1}{2}\times\frac{38}{9}=\frac{1\times 38}{2\times 9}=\frac{38}{18}=2\frac{2}{18}=2\frac{1}{9}$

$(b)(i)\frac{5}{8}\ of\ 3\frac{5}{6}=\frac{5}{8}\times\frac{23}{6}=\frac{5\times 23}{8\times 6}=\frac{115}{48}=2\frac{19}{48}$

$(ii)\frac{5}{8}\ of\ 9\frac{2}{3}=\frac{5}{8}\times\frac{29}{3}=\frac{5\times 29}{8\times 3}=\frac{145}{24}=6\frac{1}{24}$

8. Vidya and Pratap went for a picnic. Their mother gave them a water bottle that contained 5 litres of water. Vidya consumed $\frac{2}{5}$ of the water. Pratap consumed the remaining water. (i) How much water did Vidya drink? (ii) What fraction of the total quantity of water did Pratap drink?

Sol: Total water=5 litres

(i) Water drank by Vidya $=\frac{2}{5}\ of\ 5\ litres=\frac{2}{5}\times 5\ litres=\frac{10}{5}=2\ litres$

(ii) Water drank by Pratap $=5-2=3\ litres$

Fraction of water drank by Pratap $=\frac{3}{5}$

Multiplication of a Fraction by a Fraction:

Fraction × Fraction $= \frac{\text{Product of numerators}}{\text{Product of Denominators}}$; $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

(i) $\frac{1}{2} \times \frac{1}{7} = \frac{1 \times 1}{2 \times 7} = \frac{1}{14}$

(ii) $\frac{1}{5} \times \frac{1}{7} = \frac{1 \times 1}{5 \times 7} = \frac{1}{35}$

(iii) $\frac{1}{7} \times \frac{1}{2} = \frac{1 \times 1}{7 \times 2} = \frac{1}{14}$

(vi) $\frac{1}{7} \times \frac{1}{5} = \frac{1 \times 1}{7 \times 5} = \frac{1}{35}$

Exp 2: Sushant reads $\frac{1}{3}$part of a book in 1 hour. How much part of the book will he read in $2\frac{1}{5}$ hours?

Sol: The part of the book read by Sushant in 1 hour $= \frac{1}{3}$.

The part of the book read by Sushant in $2\frac{1}{5}$hour $= 2\frac{1}{5} \times \frac{1}{3} = \frac{11}{5} \times \frac{1}{3} = \frac{11 \times 1}{5 \times 3} = \frac{11}{15}$

TRY THESE

(i) $\frac{1}{3} \times \frac{4}{5} = \frac{1 \times 4}{3 \times 5} = \frac{4}{15}$ (ii) $\frac{2}{3} \times \frac{1}{5} = \frac{2 \times 1}{3 \times 5} = \frac{2}{15}$

(iii) $\frac{8}{3} \times \frac{4}{7} = \frac{8 \times 4}{3 \times 7} = \frac{32}{21}$ (vi) $\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12} = \frac{1}{2}$

Value of the Products:

For fractions $\frac{a}{b}, \frac{x}{y}$ (i) $If\ ay < bx\ then\ \frac{a}{b} < \frac{x}{y}$ and (ii) $If\ ay > bx\ then\ \frac{a}{b} > \frac{x}{y}$

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$	$\frac{8}{15} < \frac{2}{5}, \frac{8}{15} < \frac{4}{5}$	Product is less than each of the fractions
$\frac{1}{5} \times \frac{2}{7} = \frac{2}{35}$	$\frac{2}{35} < \frac{1}{5}, \frac{2}{35} < \frac{2}{7}$	Product is less than each of the fractions
$\frac{3}{5} \times \frac{7}{8} = \frac{21}{40}$	$\frac{21}{40} < \frac{3}{5}, \frac{21}{40} < \frac{7}{8}$	Product is less than each of the fractions
$\frac{2}{5} \times \frac{4}{9} = \frac{8}{45}$	$\frac{8}{45} < \frac{2}{5}, \frac{8}{45} < \frac{4}{9}$	Product is less than each of the fractions

The value of the product of two proper fractions is smaller than each of the two fractions.

$\frac{7}{3} \times \frac{5}{2} = \frac{35}{6}$	$\frac{35}{6} > \frac{7}{3}, \frac{35}{6} > \frac{5}{2}$	Product is greater than each of the fractions
$\frac{6}{5} \times \frac{4}{3} = \frac{24}{15}$	$\frac{24}{15} > \frac{6}{5}, \frac{24}{15} > \frac{4}{3}$	Product is greater than each of the fractions
$\frac{9}{2} \times \frac{7}{4} = \frac{63}{8}$	$\frac{63}{8} > \frac{9}{2}, \frac{63}{8} > \frac{7}{4}$	Product is greater than each of the fractions
$\frac{3}{2} \times \frac{8}{7} = \frac{24}{14}$	$\frac{24}{14} > \frac{3}{2}, \frac{24}{14} > \frac{8}{7}$	Product is greater than each of the fractions

The value of the product of two improper fractions is more than each of the two fractions.

$\frac{2}{3}\times\frac{7}{5}=\frac{14}{15}$	$\frac{14}{15}>\frac{2}{3},\frac{14}{15}<\frac{7}{5}$	Product is greater than proper fraction and less than improper fraction.
$\frac{6}{5}\times\frac{2}{8}=\frac{12}{40}=\frac{3}{10}$	$\frac{3}{10}<\frac{6}{5},\frac{3}{10}>\frac{2}{8}$	
$\frac{8}{3}\times\frac{4}{5}=\frac{32}{15}$	$\frac{32}{15}<\frac{8}{3},\frac{32}{15}>\frac{4}{5}$	

The value of the product of proper and improper fraction is less than proper fraction and greater than improper fraction.

EXERCISE 2.2

1. Find

$(a)\quad \frac{1}{4}of\frac{1}{4}=\frac{1}{4}\times\frac{1}{4}=\frac{1\times1}{4\times4}=\frac{1}{16}$

$(b)\quad \frac{1}{4}of\frac{3}{5}=\frac{1}{4}\times\frac{3}{5}=\frac{1\times3}{4\times5}=\frac{3}{20}$

$(c)\quad \frac{1}{4}of\frac{4}{3}=\frac{1}{4}\times\frac{4}{3}=\frac{1\times4}{4\times3}=\frac{4}{12}$

$(a)\quad \frac{1}{7}of\frac{2}{9}=\frac{1}{7}\times\frac{2}{9}=\frac{1\times2}{7\times9}=\frac{2}{63}$

$(b)\quad \frac{1}{7}of\frac{6}{5}=\frac{1}{7}\times\frac{6}{5}=\frac{1\times6}{7\times5}=\frac{6}{35}$

$(c)\quad \frac{1}{7}of\frac{3}{10}=\frac{1}{7}\times\frac{3}{10}=\frac{1\times3}{7\times10}=\frac{3}{70}$

2. Multiply and reduce to lowest form (if possible) :

$(i)\quad \frac{2}{3}\times2\frac{2}{3}=\frac{2}{3}\times\frac{8}{3}=\frac{2\times8}{3\times3}=\frac{16}{9}$

$(ii)\quad \frac{2}{7}\times\frac{7}{9}=\frac{2\times7}{7\times9}=\frac{14}{63}=\frac{2}{9}$

$(iii)\quad \frac{3}{8}\times\frac{6}{4}=\frac{3\times6}{8\times4}=\frac{18}{32}=\frac{9}{16}$

$(iv)\quad \frac{9}{5}\times\frac{3}{5}=\frac{9\times3}{5\times5}=\frac{27}{25}$

$(v)\quad \frac{1}{3}\times\frac{15}{8}=\frac{1\times15}{3\times8}=\frac{15}{24}=\frac{5}{8}$

$(vi)\quad \frac{11}{2}\times\frac{3}{10}=\frac{11\times3}{2\times10}=\frac{33}{20}$

$(vii)\quad \frac{4}{5}\times\frac{12}{7}=\frac{4\times12}{5\times7}=\frac{48}{35}$

3. Multi1ply the following fractions:

$(i)\quad \frac{2}{5}\times5\frac{1}{4}=\frac{2}{5}\times\frac{21}{4}=\frac{2\times21}{5\times4}=\frac{21}{10}$

$(ii)\quad 6\frac{2}{5}\times\frac{7}{9}=\frac{32}{5}\times\frac{7}{9}=\frac{32\times7}{5\times9}=\frac{224}{45}$

$(iii)\quad \frac{3}{2}\times5\frac{1}{3}=\frac{3}{2}\times\frac{16}{3}=\frac{3\times16}{2\times3}=\frac{8}{1}=8$

$(iv)\quad \frac{5}{6}\times2\frac{3}{7}=\frac{5}{6}\times\frac{17}{7}=\frac{5\times17}{6\times7}=\frac{85}{42}$

$(v)\quad 3\frac{2}{5}\times\frac{4}{7}=\frac{17}{5}\times\frac{4}{7}=\frac{17\times4}{5\times7}=\frac{68}{35}$

$(vi)\quad 2\frac{3}{5}\times3=\frac{13}{5}\times\frac{3}{1}=\frac{13\times3}{5\times1}=\frac{39}{5}$

$(vii)\quad 3\frac{4}{7}\times\frac{3}{5}=\frac{25}{7}\times\frac{3}{5}=\frac{25\times3}{7\times5}=\frac{15}{7}$

4. Which is greater:

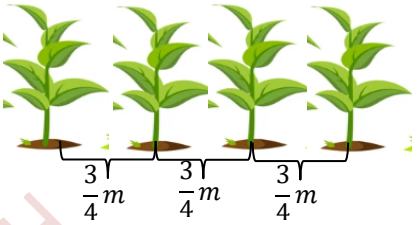
(i) $\frac{2}{7} of \frac{3}{4} = \frac{2}{7} \times \frac{3}{4} = \frac{2 \times 3}{7 \times 4} = \frac{3}{14}$; $\frac{3}{5} of \frac{5}{8} = \frac{3}{5} \times \frac{5}{8} = \frac{3 \times 5}{5 \times 8} = \frac{3}{8}$

$\frac{3}{8} > \frac{3}{14} \Rightarrow \frac{3}{5} of \frac{5}{8}$ is greater

(ii) $\frac{1}{2} of \frac{6}{7} = \frac{1}{2} \times \frac{6}{7} = \frac{1 \times 6}{2 \times 7} = \frac{3}{7}$; $\frac{2}{3} of \frac{3}{7} = \frac{2}{3} \times \frac{3}{7} = \frac{2 \times 3}{3 \times 7} = \frac{2}{7}$

$\frac{3}{7} > \frac{2}{7} \Rightarrow \frac{1}{2} of \frac{6}{7}$ is greater

5. Saili plants 4 saplings, in a row, in her garden. The distance between two adjacent saplings is $\frac{3}{4}$m. Find the distance between the first and the last sapling.



Sol: *The distance between two adjacent saplings* $= \frac{3}{4}m$

Total number of saplings=4

The distance between the first and the last sapling $= 3 \times \frac{3}{4}m = \frac{9}{4}m = 2\frac{1}{4}m$

6. Lipika reads a book for $1\frac{3}{4}$ hours everyday. She reads the entire book in 6 days. How many hours in all were required by her to read the book?

Sol: Lipika reads a book in 1 day $= 1\frac{3}{4}\ h = \frac{7}{4}\ h$

Total days=6

Total hours taken by Lipika to read the book $= 6 \times \frac{7}{4} = \frac{6 \times 7}{4} = \frac{21}{2} = 10\frac{1}{2}\ hours$

7. A car runs 16 km using 1 litre of petrol. How much distance will it cover using $2\frac{3}{4}$ litres of petrol.

Sol: Car covers the distance using 1 litre of petrol=16 km

Car covers the distance using $2\frac{3}{4}$ litres of petrol $= 2\frac{3}{4} \times 16\ km = \frac{11}{\cancel{4}} \times \cancel{16}^{4} = 11 \times 4 = 44\ km$

8. (a) (i)) Provide the number in the box☐ , such that $\frac{2}{3} \times \square = \frac{10}{30}$

(ii) The simplest form of the number obtained in ☐ is____

Sol: $\frac{2}{3} \times \frac{5}{10} = \frac{2 \times 5}{3 \times 10} = \frac{10}{30} = \frac{1}{3}$

(b) (i)) Provide the number in the box☐ , such that $\frac{3}{5} \times \square = \frac{24}{75}$

(ii) The simplest form of the number obtained in ☐ is____

Sol: $\frac{3}{5} \times \frac{8}{15} = \frac{3 \times 8}{5 \times 15} = \frac{24}{75} = \frac{8}{25}$

Reciprocal of a fraction:

The non-zero numbers whose product with each other is 1, are called the reciprocals of each other.

Swap over the Numerator and Denominator we get reciprocal of the fraction.

The reciprocal of $\frac{a}{b} = \frac{b}{a}$

THINK, DISCUSS AND WRITE

(i) **Will the reciprocal of a proper fraction be again a proper fraction?**

Sol: No, The reciprocal of a proper fraction is always an improper fraction.

$\frac{2}{7}$ is a proper fraction . Reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$ an improper fraction.

(ii) **Will the reciprocal of an improper fraction be again an improper fraction?**

Sol: No, The reciprocal of a proper fraction is always an improper fraction

$\frac{9}{5}$ is an improper fraction . Reciprocal of $\frac{9}{5} = \frac{5}{9}$ is a proper fraction

DIVISION OF FRACTIONS

To divide a whole number by any fraction, multiply that whole number by the reciprocal of that fraction.

$(i) \quad 7 \div \frac{2}{5} = 7 \times \frac{5}{2} = \frac{7 \times 5}{2} = \frac{35}{2} = 17\frac{1}{2}$

$(ii) \quad 6 \div \frac{4}{7} = 6 \times \frac{4}{7} = \frac{6 \times 4}{7} = \frac{24}{7} = 3\frac{3}{7}$

$(iii) \quad 2 \div \frac{8}{9} = 2 \times \frac{9}{8} = \frac{2 \times 9}{8} = \frac{9}{4} = 2\frac{1}{4}$

While dividing a whole number by a mixed fraction, first convert the mixed fraction into improper fraction and then solve it.

$(i) \quad 4 \div 2\frac{2}{15} = 4 \div \frac{12}{5} = 4 \times \frac{5}{12} = \frac{4 \times 5}{12} = \frac{5}{3} = 1\frac{2}{3}$

$(ii) \quad 5 \div 3\frac{1}{3} = 5 \div \frac{10}{3} = 5 \times \frac{3}{10} = \frac{5 \times 3}{10} = \frac{3}{2} = 1\frac{1}{2}$

$(iii) \quad 6 \div 5\frac{1}{43} = 6 \div \frac{16}{3} = 6 \times \frac{3}{16} = \frac{6 \times 3}{16} = \frac{9}{8} = 1\frac{1}{8}$

$(vi) \quad 7 \div 2\frac{4}{7} = 7 \div \frac{18}{7} = 7 \times \frac{7}{18} = \frac{7 \times 7}{18} = \frac{49}{18} = 2\frac{13}{18}$

Division of a Fraction by a Whole Number or another fraction

$(i)\quad \frac{3}{4}\div 3=\frac{3}{4}\div\frac{3}{1}=\frac{3}{4}\times\frac{1}{3}=\frac{3\times 1}{4\times 3}=\frac{1}{4}$

$(ii)\quad 2\frac{2}{3}\div 5=\frac{8}{3}\div\frac{5}{1}=\frac{8}{3}\times\frac{1}{5}=\frac{8\times 1}{3\times 5}=\frac{8}{15}$

$(iii)\quad \frac{1}{3}\div\frac{6}{5}=\frac{1}{3}\times\frac{5}{6}=\frac{1\times 5}{3\times 6}=\frac{5}{18}$

$(iv)\quad \frac{8}{5}\div\frac{2}{3}=\frac{8}{5}\times\frac{3}{2}=\frac{8\times 3}{5\times 2}=\frac{12}{5}=2\frac{2}{5}$

$(vi)\quad \frac{3}{5}\div\frac{1}{2}=\frac{3}{5}\times\frac{2}{1}=\frac{3\times 2}{5\times 1}=\frac{6}{5}=1\frac{1}{5}$

$(vii)\quad \frac{1}{2}\div\frac{3}{5}=\frac{1}{2}\times\frac{5}{3}=\frac{1\times 5}{2\times 3}=\frac{5}{6}$

$(viii)\quad 2\frac{1}{2}\div\frac{3}{5}=\frac{5}{2}\times\frac{5}{3}=\frac{5\times 5}{2\times 3}=\frac{25}{6}=4\frac{1}{6}$

$(ix)\quad 5\frac{1}{6}\div\frac{9}{2}=\frac{31}{6}\div\frac{9}{2}=\frac{31}{6}\times\frac{2}{9}=\frac{31\times 2}{6\times 9}=\frac{31}{27}=1\frac{4}{27}$

EXERCISE 2.3

1. Find

$(i)\quad 12\div\frac{3}{4}=\frac{12}{1}\div\frac{3}{4}=\frac{12}{1}\times\frac{4}{3}=\frac{12\times 4}{1\times 3}=\frac{16}{1}=16$

$(ii)\quad 14\div\frac{5}{6}=\frac{14}{1}\div\frac{5}{6}=\frac{14}{1}\times\frac{6}{5}=\frac{14\times 6}{1\times 5}=\frac{84}{5}=16\frac{4}{5}$

$(iii)\quad 8\div\frac{7}{3}=\frac{8}{1}\div\frac{7}{3}=\frac{8}{1}\times\frac{3}{7}=\frac{8\times 3}{1\times 7}=\frac{24}{7}=3\frac{3}{7}$

$(iv)\quad 4\div\frac{8}{3}=\frac{4}{1}\div\frac{8}{3}=\frac{4}{1}\times\frac{3}{8}=\frac{4\times 3}{1\times 8}=\frac{3}{2}=1\frac{1}{2}$

$(v)\quad 3\div 2\frac{1}{3}=\frac{3}{1}\div\frac{7}{3}=\frac{3}{1}\times\frac{3}{7}=\frac{3\times 3}{1\times 7}=\frac{9}{7}=1\frac{2}{7}$

$(vi)\quad 5\div 3\frac{4}{7}=\frac{5}{1}\div\frac{25}{7}=\frac{5}{1}\times\frac{7}{25}=\frac{5\times 7}{1\times 25}=\frac{7}{5}=1\frac{2}{5}$

2. Find the reciprocal of each of the following fractions. Classify the reciprocals as proper fractions, improper fractions and whole numbers.

Sol: (i) Reciprocal of $\frac{3}{7}=\frac{7}{3}$ → Improper fraction

(ii) Reciprocal of $\frac{5}{8}=\frac{8}{5}$ → Improper fraction

(iii) Reciprocal of $\frac{9}{7} = \frac{7}{9}$ → Proper fraction

(iv) Reciprocal of $\frac{6}{5} = \frac{5}{6}$ → Proper fraction

(v) Reciprocal of $\frac{12}{7} = \frac{7}{12}$ → Proper fraction

(vi) Reciprocal of $\frac{1}{8} = 8$ → Whole number

(vii) Reciprocal of $\frac{1}{11} = 11$ → Whole number

3. Find:

(i) $\frac{7}{3} \div 2 = \frac{7}{3} \div \frac{2}{1} = \frac{7}{3} \times \frac{1}{2} = \frac{7 \times 1}{3 \times 2} = \frac{7}{6} = 1\frac{1}{6}$

(ii) $\frac{4}{9} \div 5 = \frac{4}{9} \div \frac{5}{1} = \frac{4}{9} \times \frac{1}{5} = \frac{4 \times 1}{9 \times 5} = \frac{4}{45}$

(iii) $\frac{6}{13} \div 7 = \frac{6}{13} \div \frac{7}{1} = \frac{6}{13} \times \frac{1}{7} = \frac{6 \times 1}{13 \times 7} = \frac{6}{91}$

(iv) $4\frac{1}{3} \div 3 = \frac{13}{3} \div \frac{3}{1} = \frac{13}{3} \times \frac{1}{3} = \frac{13 \times 1}{3 \times 3} = \frac{13}{9}$

(v) $3\frac{1}{2} \div 4 = \frac{7}{2} \div \frac{4}{1} = \frac{7}{2} \times \frac{1}{4} = \frac{7 \times 1}{2 \times 4} = \frac{7}{8}$

(vi) $4\frac{3}{7} \div 7 = \frac{31}{7} \div \frac{7}{1} = \frac{31}{7} \times \frac{1}{7} = \frac{31 \times 1}{7 \times 7} = \frac{31}{49}$

4. Find:

(i) $\frac{2}{5} \div \frac{1}{2} = \frac{2}{5} \times \frac{2}{1} = \frac{2 \times 2}{5 \times 1} = \frac{4}{5}$

(ii) $\frac{4}{9} \div \frac{2}{3} = \frac{4}{9} \times \frac{3}{2} = \frac{4 \times 3}{9 \times 2} = \frac{2}{3}$

(iii) $\frac{3}{7} \div \frac{8}{7} = \frac{3}{7} \times \frac{7}{8} = \frac{3 \times 7}{7 \times 8} = \frac{3}{8}$

(iv) $2\frac{1}{3} \div \frac{3}{5} = \frac{7}{3} \div \frac{3}{5} = \frac{7}{3} \times \frac{5}{3} = \frac{7 \times 5}{3 \times 3} = \frac{35}{9} = 3\frac{7}{9}$

(v) $3\frac{1}{2} \div \frac{8}{3} = \frac{7}{2} \div \frac{8}{3} = \frac{7}{2} \times \frac{3}{8} = \frac{7 \times 3}{2 \times 8} = \frac{21}{16} = 1\frac{5}{16}$

(vi) $\frac{2}{5} \div 1\frac{1}{2} = \frac{2}{5} \div \frac{3}{2} = \frac{2}{5} \times \frac{2}{3} = \frac{2 \times 2}{5 \times 3} = \frac{4}{15}$

$(vii)\ \ 3\frac{1}{5} \div 1\frac{2}{3} = \frac{16}{5} \div \frac{5}{3} = \frac{16}{5} \times \frac{3}{5} = \frac{16 \times 3}{5 \times 5} = \frac{48}{25} = 1\frac{23}{25}$

$(viii)\ 2\frac{1}{5} \div 1\frac{1}{5} = \frac{8}{5} \div \frac{6}{5} = \frac{8}{5} \times \frac{5}{6} = \frac{8 \times 5}{5 \times 6} = \frac{4}{3} = 1\frac{1}{3}$

Decimal numbers:

A decimal is another way of expressing a fraction. The dot or the point between the two digits is called the decimal point. Number of digits after the decimal point is called the number of decimal places.

$$\text{Whole number part} \rightarrow 32.564 \leftarrow \text{Decimal part}$$
($\downarrow$ Decimal point)

The number of digits after the decimal point in the decimal numeral is equal to the number of zeros after 1 in the denominator of the corresponding common fraction.

$$\frac{4}{10} = 0.4, \quad \frac{2}{100} = 0.02, \quad \frac{37}{1000} = 0.037\,, \quad \frac{426}{100} = 4.26\,, \quad \frac{527}{10} = 52.7\,, \quad \frac{784}{1000} = 0.784$$

Multiplication Decimals:

The number of decimals digits in the product of any two decimal numbers is equal to the sum of decimal digits that are multiplied.

TRY THESE

1. Find:

$(i)\ \ 2.7 \times 4 = 10.8$

$(ii)\ \ 1.8 \times 1.2 = 2.16$

$(iii)\ \ 2.3 \times 4.35 = 10.005$

2. Arrange the products obtained in (1) in descending order.

Sol: The products in descending order: , 10.8, 10.005, 2.16,

Exp 3: The side of an equilateral triangle is 3.5 cm. Find its perimeter.

Sol: Length of side=3.5 cm

Perimeter of equilateral triangle =3×side=3×3.5cm=10.5 cm

Exp 4: The length of a rectangle is 7.1 cm and its breadth is 2.5 cm. What is the area of the rectangle?

Sol: Length(l) = 7.1 cm and Breadth(b) = 2.5 cm

Area of the rectangle = l × b = 7.1 × 2.5 cm^2 = 17.75 cm^2

$$\begin{array}{r} 7.1 \\ \times\ 2.5 \\ \hline 355 \\ 142X \\ \hline 17.75 \end{array}$$

Multiplication of Decimal Numbers by 10, 100 and 1000:

when a decimal number is multiplied by 10, 100 or 1000, the digits in the product are same as in the decimal number but the decimal point in the product is shifted to the right by as, many of places as there are zeros over one.

$1.76 \times 10 = 176$	$12.356 \times 100 = 1235.6$	$0.3 \times 10 = 3$
$2.35 \times 10 = 23.5$	$1.76 \times 1000 = 1760$	$1.2 \times 100 = 120$
$12.356 \times 10 = 123.56$	$2.35 \times 1000 = 2350$	$56.3 \times 1000 = 56300$
$1.76 \times 100 = 176$	$12.356 \times 1000 = 12356$	
$2.35 \times 100 = 235$	$0.5 \times 1000 = 500$	

EXERCISE 2.4

1. Find

(i) $0.2 \times 6 = 1.2$	(iv) $20.1 \times 4 = 80.4$	(vi) $2 \times 0.86 = 1.72$
(ii) $8 \times 4.6 = 36.8$	(v) $0.05 \times 7 = 0.35$	
(iii) $2.71 \times 5 = 13.55$	(vi) $211.02 \times 4 = 844.08$	

2. Find the area of rectangle whose length is 5.7cm and breadth is 3 cm.

Sol: Length(l) = 5.7 cm and Breadth(b) = 3 cm

Area of the rectangle = $l \times b = 5.7 \times 3\ cm^2 = 17.1\ cm^2$

3. Find:

(i) $1.3 \times 10 = 13$	(v) $31.1 \times 100 = 3110$	(ix) $0.5 \times 10 = 5$
(ii) $36.8 \times 10 = 368$	(vi) $156.1 \times 100 = 15610$	(x) $0.08 \times 10 = 0.8$
(iii) $153.7 \times 10 = 1537$	(vii) $3.62 \times 100 = 362$	(xi) $0.9 \times 100 = 90$
(iv) $168.07 \times 10 = 1680.7$	(viii) $43.07 \times 100 = 4307$	(xii) $0.03 \times 1000 = 30$

4. A two-wheeler covers a distance of 55.3 km in one litre of petrol. How much distance will it cover in 10 litres of petrol?

Sol: The distance covers by a two wheeler in 1 litre of petrol=55.3 km

The distance covers by a two wheeler in 10 litre of petrol=10×55.3 km=553 km

5. Find:

(i) $2.5 \times 0.3 = 0.75$	(v) $0.5 \times 0.05 = 0.025$	(ix) $101.01 \times 0.01 = 1.0101$
(ii) $0.1 \times 51.7 = 517$	(vi) $11.2 \times 0.15 = 1.68$	(x) $100.01 \times 1.1 = 110.011$
(iii) $0.2 \times 316.8 = 63.36$	(vii) $1.07 \times 0.02 = 0.0214$	
(iv) $1.3 \times 3.1 = 4.03$	(viii) $10.05 \times 1.05 = 10.5525$	

DIVISION OF DECIMAL NUMBERS:

Division by 10, 100 and 1000

While dividing a number by 10, 100 or 1000, the digits of the number and the quotient are same but the decimal point in the quotient shifts to the left by as many places as there are zeros over.

$31.5 \div 10 = 3.15$	$31.5 \div 100 = 0.315$	$31.5 \div 1000 = 0.0315$
$231.5 \div 10 = 23.15$	$231.5 \div 10 = 23.15$	$231.5 \div 1000 = 0.2315$
$1.5 \div 10 = 0.15$	$1.5 \div 100 = 0.015$	$1.5 \div 1000 = 0.0015$
$29.36 \div 10 = 2.936$	$29.36 \div 100 = 0.2936$	$29.36 \div 1000 = 0.02936$

Try these

(i) $235.4 \div 10 = 23.54$

(ii) $235.4 \div 100 = 2.354$

(iii) $235.4 \div 1000 = 0.2354$

Division of a Decimal Number by a Whole Number:

(i) $6.4 \div 2 = 3.2$

(ii) $35.7 \div 3 = 11.9$

(iii) $25.5 \div 3 = 8.5$

(iv) $43.15 \div 5 = 8.63$

(v) $82.44 \div 6 = 13.74$

(vi) $15.5 \div 5 = 3.1$

(vii) $126.35 \div 7 = 18.05$

$(viii)$ $12.96 \div 4 = 3.24$

$$\begin{array}{r} 3.2 \\ 2\overline{)6.4} \\ 6 \\ \hline 4 \\ 4 \\ \hline 0 \end{array} \qquad \begin{array}{r} 11.9 \\ 3\overline{)35.7} \\ 3 \\ \hline 5 \\ 3 \\ \hline 27 \\ 27 \\ \hline 0 \end{array} \qquad \begin{array}{r} 8.5 \\ 3\overline{)25.5} \\ 24 \\ \hline 15 \\ 15 \\ \hline 0 \end{array} \qquad \begin{array}{r} 8.63 \\ 5\overline{)43.15} \\ 40 \\ \hline 31 \\ 30 \\ \hline 15 \\ 15 \\ \hline 0 \end{array} \qquad \begin{array}{r} 13.74 \\ 6\overline{)82.44} \\ 6 \\ \hline 22 \\ 18 \\ \hline 44 \\ 42 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 3.1 \\ 5\overline{)15.5} \\ 15 \\ \hline 05 \\ 5 \\ \hline 0 \end{array} \qquad \begin{array}{r} 18.05 \\ 7\overline{)126.35} \\ 7 \\ \hline 56 \\ 56 \\ \hline 03 \\ 0 \\ \hline 35 \\ 35 \\ \hline 0 \end{array} \qquad \begin{array}{r} 3.24 \\ 4\overline{)12.96} \\ 12 \\ \hline 09 \\ 8 \\ \hline 16 \\ 16 \\ \hline 0 \end{array}$$

Exp 5: Find the average of 4.2, 3.8 and 7.6

Sol: $\text{Average} = \dfrac{\text{Sum of observations}}{\text{Number of observations}} = \dfrac{4.2 + 3.8 + 7.6}{3} = \dfrac{15.6}{3} = 5.2$

Division of a Decimal Number by another Decimal Number:

Ex: $25.5 \div 0.5 = \dfrac{25.5 \times 10}{0.5 \times 10} = \dfrac{255}{5} = 51$

Ex: $22.5 \div 1.5 = \dfrac{22.5 \times 10}{1.5 \times 10} = \dfrac{225}{15} = 15$

TRY THESE

(i) $\dfrac{7.75}{0.25} = \dfrac{7.75 \times 100}{0.25 \times 100} = \dfrac{775}{25} = 31$

$(ii)\ \dfrac{42.8}{0.02} = \dfrac{42.8 \times 100}{0.02 \times 100} = \dfrac{4280}{2} = 2140$

$(iii)\ \dfrac{5.6}{1.4} = \dfrac{5.6 \times 10}{1.4 \times 10} = \dfrac{56}{14} = 4$

Exp 6: Each side of a regular polygon is 2.5 cm in length. The perimeter of the polygon is 12.5cm. How many sides does the polygon have?

Sol: Length of each side of polygon = 2.5 cm

Number of sides of the polygon $= \dfrac{\text{perimeter of the polygon}}{\text{Length of each side of polygon}} = \dfrac{12.5}{2.5} = \dfrac{125}{25} = 5$

Exp7 :A car covers a distance of 89.1 km in 2.2 hours. What is the average distance covered by it in 1 hour?

Sol: Distance covered in 2.2 hours=89.1 km

Distance covered in 1 hour $= \dfrac{89.1}{2.2} = \dfrac{891}{22} = \dfrac{81}{2} = 40.5$ km

EXERCISE 2.5

1. Find:

(i) $0.4 \div 2 = 0.2$

(ii) $0.35 \div 5 = 0.07$

(iii) $2.48 \div 4 = 0.62$

(iv) $65.4 \div 6 = 10.9$

(v) $651.2 \div 4 = 162.8$

(vi) $14.49 \div 7 = 2.07$

(vii) $3.96 \div 4 = 0.99$

(viii) $0.80 \div 5 = 0.16$

2. **Find**:

(i) $4.8 \div 10 = 0.48$

(ii) $52.5 \div 10 = 52.5$

(iii) $0.7 \div 10 = 0.07$

(iv) $33.1 \div 10 = 3.31$

(v) $272.23 \div 10 = 27.223$

(vi) $0.56 \div 10 = 0.056$

(vii) $3.97 \div 10 = 0.397$

3. Find:

(i) $2.7 \div 100 = 0.027$

(ii) $0.3 \div 100 = 0.003$

(iii) $0.78 \div 100 = 0.0078$

(iv) $432.6 \div 100 = 4.326$

(v) $23.6 \div 100 = 0.236$

(vi) $98.53 \div 100 = 0.9853$

4. Find

(i) $7.9 \div 1000 = 0.0079$

(ii) $26.3 \div 1000 = 0.0263$

(iii) $38.53 \div 1000 = 0.03853$

(iv) $128.9 \div 1000 = 0.1289$

(v) $0.5 \div 1000 = 0.005$

5. Find:

$(i) \quad 7 \div 3.5 = \frac{7 \times 10}{3.5 \times 10} = \frac{70}{35} = 2$

$(ii)\ 36 \div 0.2 = \frac{36 \times 10}{0.2 \times 10} = \frac{360}{2} = 180$

$(iii)\ 3.25 \div 0.5 = \frac{3.25 \times 10}{0.5 \times 10} = \frac{32.5}{5} = 6.5$

$(iv)\ 30.94 \div 0.7 = \frac{30.94 \times 10}{0.7 \times 10} = \frac{309.4}{7} = 44.2$

$(v)\ 0.5 \div 0.25 = \frac{0.5 \times 100}{0.25 \times 100} = \frac{50}{25} = 2$

$(vi)\ 7.75 \div 0.25 = \frac{7.75 \times 100}{0.25 \times 100} = \frac{775}{25} = 31$

$(vii)\ 76.5 \div 0.15 = \frac{76.5 \times 100}{0.15 \times 100} = \frac{7650}{15} = 510$

$(viii)\ 37.8 \div 1.4 = \frac{37.8 \times 10}{1.4 \times 10} = \frac{378}{14} = 27$

$(ix)\ 2.73 \div 1.3 = \frac{2.73 \times 10}{1.3 \times 10} = \frac{27.3}{13} = 2.1$

6. A vehicle covers a distance of 43.2 km in 2.4 litres of petrol. How much distance will it cover in one litre of petrol?

Sol: Vehicle covers distance in 2.4 litres of petrol=43.2 km

Vehicle covers distance in 1 litre of petrol $= \frac{43.2}{2.4} = \frac{432}{24} = 18$

CHAPTER 3

VII-MATHEMATICS-NCERT

3. DATA HANDLING (NOTES)

PREPARED BY: BALABHADRA SURESH

1. A data is a collection of numbers gathered to give some information.
2. A pictograph represents data through pictures of objects.
3. To get a particular information from the given data quickly, the data can be arranged in a tabular form using tally marks

ARITHMETIC MEAN

$$\text{Arithmetic mean (Average)} = \frac{\text{Sum of all observations}}{\text{number of observations}}$$

Mean lies in between the **greatest** and the **smallest** observations.

Exp 1:Ashish studies for 4 hours, 5 hours and 3 hours respectively on three consecutive days. How many hours does he study daily on an average?

Sol: $\text{average time} = \frac{\text{Total hours}}{\text{Total days}} = \frac{4+5+3}{3} = \frac{12}{3} = 4\ hours$

Exp 2:A batsman scored the following number of runs in six innings: 36, 35, 50, 46, 60, 55

Sol: $Mean = \frac{\text{Sum of all observations}}{\text{number of observations}} = \frac{36+35+50+46+60+55}{6} = \frac{282}{6} = 47$

The mean runs scored in an inning are 47

1. **Find at least 5 numbers between $\frac{1}{2}$ *and* $\frac{1}{3}$**

Sol: A number between $\boldsymbol{x}$ *and* $\boldsymbol{y} = \frac{1}{2}(x + y)$

$$\text{A number between } \frac{1}{2} \text{ and } \frac{1}{3} = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{3}\right) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$$

$$\text{A number between } \frac{1}{2} \text{ and } \frac{5}{12} = \frac{1}{2}\left(\frac{1}{2} + \frac{5}{12}\right) = \frac{1}{2} \times \frac{11}{12} = \frac{11}{24}$$

$$\text{A number between } \frac{1}{2} \text{ and } \frac{11}{24} = \frac{1}{2}\left(\frac{1}{2} + \frac{11}{24}\right) = \frac{1}{2} \times \frac{23}{24} = \frac{23}{48}$$

$$\text{A number between } \frac{5}{12} \text{ and } \frac{1}{3} = \frac{1}{2}\left(\frac{5}{12} + \frac{1}{3}\right) = \frac{1}{2} \times \frac{9}{12} = \frac{9}{24}$$

A number between $\frac{9}{24}$ *and* $\frac{1}{3} = \frac{1}{2}\left(\frac{9}{24} + \frac{1}{3}\right) = \frac{1}{2} \times \frac{17}{24} = \frac{17}{48}$

Range:

Range = Highest observation − Lowest observation

Exp 3: The ages in years of 10 teachers of a school are: 32, 41, 28, 54, 35, 26, 23, 33, 38, 40

(i) What is the age of the oldest teacher and that of the youngest teacher?

Sol: The age of the oldest teacher =54 years ; The youngest teacher=23 years

(ii) What is the range of the ages of the teachers?

Sol: Range of the ages of the teachers = (54 – 23) years = 31 years

(iii) What is the mean age of these teachers?

Sol: Mean $= \frac{\text{Sum of all observations}}{\text{number of observations}} = \frac{23 + 26 + 28 + 32 + 33 + 35 + 38 + 40 + 41 + 54}{10}$

$= \frac{350}{10} = 35$

Mean age of the teachers=35 years.

EXERCISE 3.1

1. Find the range of heights of any ten students of your class.

Sol:

2. Organise the following marks in a class assessment, in a tabular form.

4, 6, 7, 5, 3, 5, 4, 5, 2, 6, 2, 5, 1, 9, 6, 5, 8, 4, 6, 7

(i) Which number is the highest? (ii) Which number is the lowest? (iii) What is the range of the data? (iv) Find the arithmetic mean.

Sol:

Marks	Tally Marks	Frequency
1	|	1
2	||	2
3	|	1
4	|||	3
5	~~||||~~	5

6	||||	4
7	||	2
8	|	1
9	|	1

3. Find the mean of the first five whole numbers.

Sol: First 5 whole numbers : 0,1,2,3,4

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{number of observations}} = \frac{0+1+2+3+4}{5} = \frac{10}{5} = 2$$

4. A cricketer scores the following runs in eight innings: 58, 76, 40, 35, 46, 45, 0, 100 Find the mean score.

Sol: $\text{Mean} = \frac{\text{Sum of all observations}}{\text{number of observations}} = \frac{58+76+40+35+46+45+0+100}{8} = \frac{400}{8} = 50$

5. Following table shows the points of each player scored in four games:

Player	Game 1	Game 2	Game 3	Game 4
A	14	16	10	10
B	0	8	6	4
C	8	11	Did not play	13

Now answer the following questions: (i) Find the mean to determine A's average number of points scored per game. (ii) To find the mean number of points per game for C, would you divide the total points by 3 or by 4? Why? (iii) B played in all the four games. How would you find the mean? (iv) Who is the best performer?

Sol: (i) Mean score of A $= \frac{\text{Sum of all observations}}{\text{number of observations}} = \frac{14+16+10+10}{4} = \frac{50}{4} = 12.5$

(ii) Player C played only three games. So, we divide the total points by 3

(iii) Mean score of B $= \frac{\text{Sum of all observations}}{\text{number of observations}} = \frac{0+8+6+4}{4} = \frac{18}{4} = 4.5$

(iv) Mean score of C $= \frac{\text{Sum of all observations}}{\text{number of observations}} = \frac{8+11+13}{3} = \frac{32}{3} = 10.67$

The mean score of A is the highest of all three players. So, A is best performer.

6. The marks (out of 100) obtained by a group of students in a science test are 85, 76, 90, 85, 39, 48, 56, 95, 81 and 75. Find the:

(i) Highest and the lowest marks obtained by the students.

(ii) Range of the marks obtained.

(iii) Mean marks obtained by the group.

Sol: (i) Highest marks=95 and Lowest marks=39

(ii) Range= Highest marks- Lowest marks=95 -39=56.

(iii)Mean marks $= \frac{Sum\ of\ marks}{number\ of\ students}$

$$= \frac{85 + 76 + 90 + 85 + 39 + 48 + 56 + 95 + 81 + 75}{10}$$

$$= \frac{730}{10} = 73$$

7. **The enrolment in a school during six consecutive years was as follows:**

1555, 1670, 1750, 2013, 2540, 2820 Find the mean enrolment of the school for this period.

Sol: Mean enrolment $= \frac{\text{Sum of enrollments}}{\text{Number of years}}$

$$= \frac{1555 + 1670 + 1750 + 2013 + 2540 + 2820}{6}$$

$$= \frac{12348}{6} = 2058$$

8. **The rainfall (in mm) in a city on 7 days of a certain week was recorded as follows:**

Day	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
Rainfall (in mm)	0.0	12.2	2.1	0.0	20.5	5.5	1.0

(i) Find the range of the rainfall in the above data.

(ii) Find the mean rainfall for the week.

(iii) On how many days was the rainfall less than the mean rainfall?

Sol: (i) Range of the rainfall=Highest value-Lowest value=20.5-0.0=20.5 mm

(ii)Mean rainfall $= \frac{\text{Sum of rainfalls}}{\text{Number of days}} = \frac{0.0 + 12.2 + 2.1 + 0.0 + 20.5 + 5.5 + 1.0}{7}$

$$= \frac{41.3}{7} = 5.9\ mm$$

(*iii*) 5 days (Mon, Wed, Thurs, Sat, Sun)

9. The heights of 10 girls were measured in cm and the results are as follows: 135, 150, 139, 128, 151, 132, 146, 149, 143, 141.

(i) What is the height of the tallest girl? (ii) What is the height of the shortest girl? (iii) What is the range of the data? (iv) What is the mean height of the girls? (v) How many girls have heights more than the mean height.

Sol: (i) The height of the tallest girl=151 cm

(ii) The height of the shortest girl=128 cm.

(iii) Range=Highest value-Lowest value=151-128=23 cm

(iv) Mean height of the girl $= \frac{Sum\ of\ heights}{Number\ of\ girls}$

$$= \frac{135 + 150 + 139 + 128 + 151 + 132 + 146 + 149 + 143 + 141}{10} = \frac{1414}{10} = 14.14\ cm$$

(v) 5 girls

Mode

The mode of a set of observations is the observation that occurs most often.

Exp4: Find the mode of the given set of numbers: 1, 1, 2, 4, 3, 2, 1, 2, 2, 4.

Sol: Arranging the numbers with same values together, we get 1, 1, 1, **2, 2, 2, 2**, 3, 4, 4

Mode of this data = 2

Try These

(i) Find the mode of 2, 6, 5, 3, 0, 3, 4, 3, 2, 4, 5, 2, 4.

Sol: Arranging the numbers in order: 0,2,2,2,3,3,3,4,4,4,5,5,6

2,3 and 4 occurs most frequently . So, mode of the data is 2,3 and 4

(ii) Find the mode of 2, 14, 16, 12, 14, 14, 16, 14, 10, 14, 18, 14.

Sol: Arranging the numbers:2,10,12,14,14,14,14,14,14,16,16,18

Mode of the data =14

Exp 5: Following are the margins of victory in the football matches of a league

1, 3, 2, 5, 1, 4, 6, 2, 5, 2, 2, 2, 4, 1, 2, 3, 1, 1, 2, 3, 2, 6, 4, 3, 2, 1, 1, 4, 2, 1, 5, 3, 3, 2, 3, 2, 4, 2, 1, 2

Sol:

Margins of Victory	Tally Bars	Number of Matches
1	𝍸 ||||	9
2	𝍸 𝍸 ||||	14
3	𝍸 ||	7
4	𝍸	5
5	|||	3
6	||	2
	Total	**40**

Exp 6: Find the mode of the numbers: 2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 8

Sol: Here, 2 and 5 both occur three times.

Mode of the data is 2 and 5.

TRY THESE

1. **Find the mode of the following data**

   12, 14, 12, 16, 15, 13, 14, 18, 19, 12, 14, 15, 16, 15, 16, 16, 15, 17, 13, 16, 16, 15, 15, 13, 15, 17, 15, 14, 15, 13, 15, 14

Sol:

Data	Tally marks	Frequency
12	|||	3
13	||||	4
14	𝍸	5
15	𝍸 𝍸	10
16	𝍸 |	6
17	||	2
18	|	1
19	|	1
	Total	32

15 occurs highest number of times (10 times)

Mode of the data=15

2. **Heights (in cm) of 25 children are given below:**

168, 165, 163, 160, 163, 161, 162, 164, 163, 162, 164, 163, 160, 163, 160, 165, 163, 162, 163, 164, 163, 160, 165, 163, 162

What is the mode of their heights? What do we understand by mode here?

Sol:

Height of the children(in cm)	Tally marks	Frequency
160	|||	3
161	|	1
162	||||	4
163	𝍸 | 𝍸	10
164	|||	3
165	|||	3
168	|	1
	total	25

Highest frequency is 10. Corresponding height is 163 cm

Mode=163 cm

MEDIAN

Given data, arranged in ascending or descending order, the median gives us the middle observation.

Exp7:Find the median of the data: 24, 36, 46, 17, 18, 25, 35.

Sol : Data in ascending order: 17, 18, 24, **25**, 35, 36, 46

Median=25

EXERCISE 3.2

1. **The scores in mathematics test (out of 25) of 15 students is as follows**

19, 25, 23, 20, 9, 20, 15, 10, 5, 16, 25, 20, 24, 12, 20 Find the mode and median of this data. Are they same?

Sol: Arranging scores in ascending order:

5,9,10,12,15,16,19,20,20,20,20,23,24,25,25

The score 20 occurs most frequently.

Mode=20

Total observations =15

Median=Middle observation= 8^{th} observation

Median=20

Yes, The mode and median are same

2. **The runs scored in a cricket match by 11 players is as follows:**
   6, 15, 120, 50, 100, 80, 10, 15, 8, 10, 15 Find the mean, mode and median of this data. Are the three same?

Sol: $\text{Mean} = \frac{\text{Sum of all observations}}{\text{number of observations}}$

$$= \frac{6+15+120+50+100+80+10+15+8+10+15}{11} = \frac{429}{11} = 39$$

Mean = 39

Arranging the scores in ascending order: 6,8,10,15,15,15,50,80,100,120.

The score 15 occurs most frequently

Mode=15

Number of observations=11

Median=Middle most observation=6^{th} observation=15

No, the three are not same.

3. **The weights (in kg.) of 15 students of a class are: 38, 42, 35, 37, 45, 50, 32, 43, 43, 40, 36, 38, 43, 38, 47**
   (i) Find the mode and median of this data. (ii) Is there more than one mode?

Sol: Arranging the weights in ascending order:

32,35,36,37,38,38,38,40,42,43,43,43,45,47,50

The heights 38 kg and 43 kg occur most frequently

Mode= 38 and 43

Total observations=15

Median= 8^{th} observation=40

(ii) Yes , there are two modes

4. Find the mode and median of the data: 13, 16, 12, 14, 19, 12, 14, 13, 14

Sol: The data in ascending order:12,12,13,13,14,14,14,16,19.

14 occurs most frequently

Mode=14

Total observations=9

Median=5^{th} observation=14

5. Tell whether the statement is true or false:

(i) The mode is always one of the numbers in a data.

Sol: True

(ii) The mean is one of the numbers in a data.

Sol: False

(iii) The median is always one of the numbers in a data.

Sol: True

(iv) The data 6, 4, 3, 8, 9, 12, 13, 9 has mean 9.

Sol: False

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{number of observations}} = \frac{6+4+3+8+9+12+13+9}{8} = \frac{64}{8} = 8$$

Bar Graphs

Exp 8: Two hundred students of 6th and 7th classes were asked to name their favourite colour so as to decide upon what should be the colour of their school building. The results are shown in the following table. Represent the given data on a bar graph.

Favourite Colour	Red	Green	Blue	Yellow	Orange
Number of Students	43	19	55	49	34

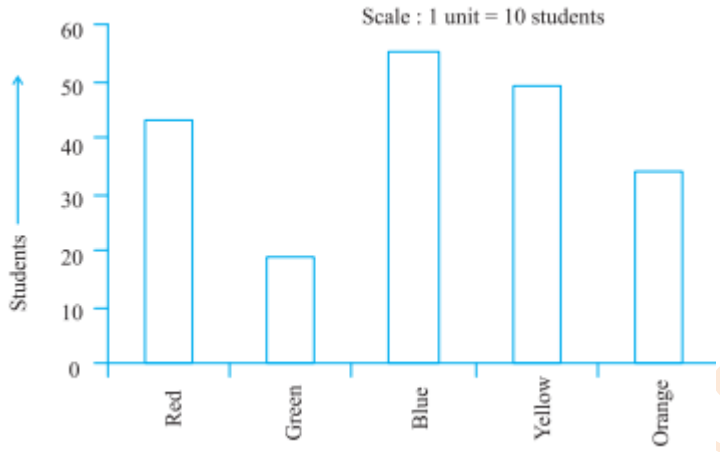
Answer the following questions with the help of the bar graph:

(i) Which is the most preferred colour and which is the least preferred? (ii) How many colours are there in all? What are they?

Sol: (i) The most preferred colour=Blue

The least preferred colour=Green

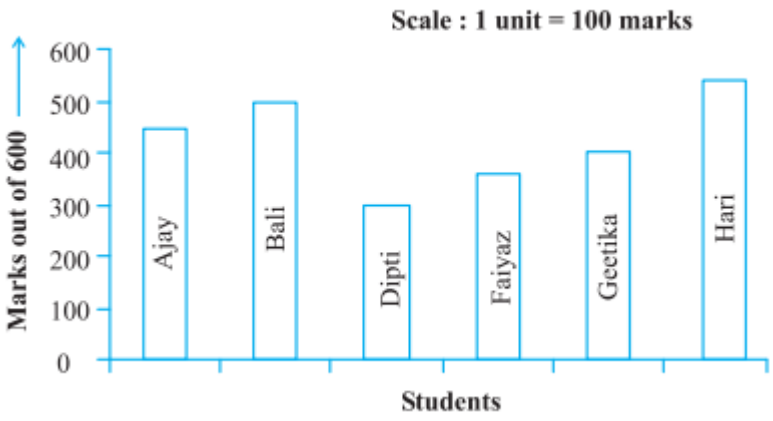
(ii) There are 5 colours in all. They are red, green, blue, yellow ,orange.



Exp 9: **Following data gives total marks (out of 600) obtained by six children of a particular class. Represent the data on a bar graph.**

Students	Ajay	Bali	Dipti	Faiyaz	Geetika	Hari
Marks Obtained	450	500	300	360	400	540

Sol:



(i) On horizontal axis take student names. On vertical axis take marks

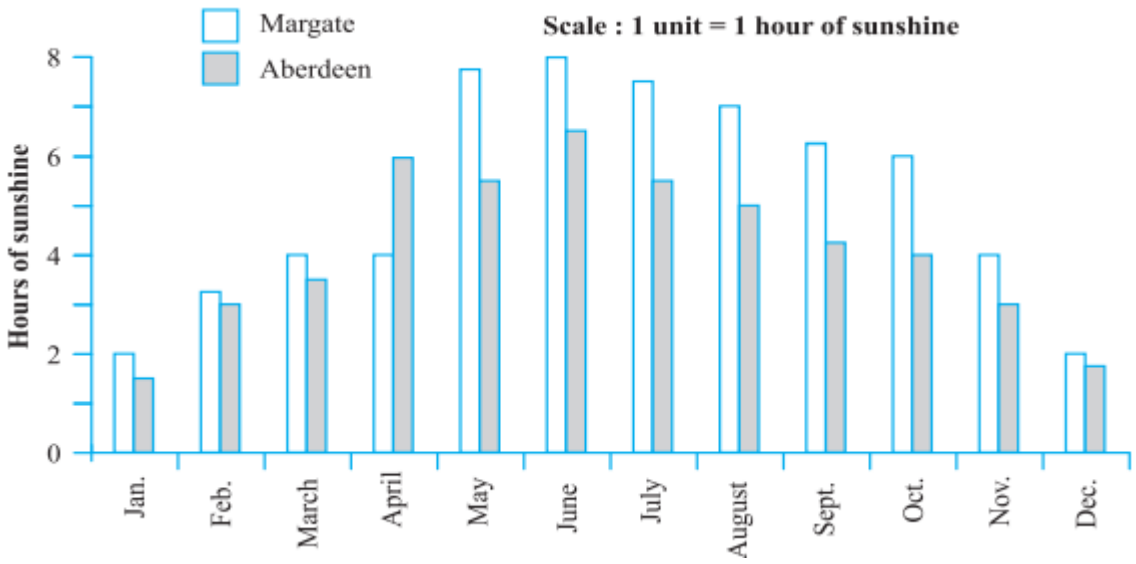
(ii) On vertical axis 1 unit=100 marks

Drawing double bar graph

In Margate												
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Average hours of Sunshine	2	$3\frac{1}{4}$	4	4	$7\frac{3}{4}$	8	$7\frac{1}{2}$	7	$6\frac{1}{4}$	6	4	2
In Aberdeen												
Average hours of Sunshine	$1\frac{1}{2}$	3	$3\frac{1}{2}$	6	$5\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{1}{2}$	5	$4\frac{1}{2}$	4	3	$1\frac{3}{4}$

By drawing individual bar graphs you could answer questions like

Sol:

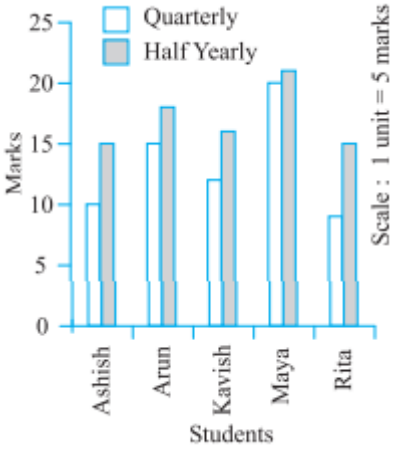


(i) In which month does each city has maximum sunlight?

Sol: June

(ii) In which months does each city has minimum sunlight?

Sol: January



Exp 10 :A mathematics teacher wants to see, whether the new technique of teaching she applied after quarterly test was effective or

not. She takes the scores of the 5 weakest children in the quarterly test (out of 25) and in the half yearly test (out of 25)

Students	Ashish	Arun	Kavish	Maya	Rita
Quarterly	10	15	12	20	9
Half yearly	15	18	16	21	15

Sol: She draws the adjoining double bar graph and finds a marked improvement in most of the students, the teacher decides that she should continue to use the new technique of teaching.

Try These

1. The bar graph (Fig 3.2) shows the result of a survey to test water resistant watches made by different companies. Each of these companies claimed that their watches were water resistant. After a test the above results were revealed.

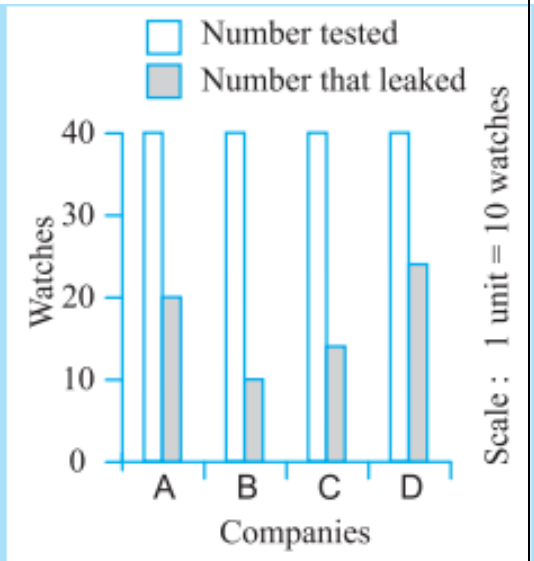


Fig 3.2

(a) Can you work out a fraction of the number of watches that leaked to the number tested for each company?

Sol:

Company	Tested	Leaked	fraction
A	40	20	$\frac{20}{40}=\frac{1}{2}$
B	40	10	$\frac{10}{40}=\frac{1}{4}$
C	40	15	$\frac{15}{40}=\frac{3}{8}$
D	40	25	$\frac{25}{40}=\frac{5}{8}$

(b) Could you tell on this basis which company has better watches?

Sol: From the above 10/40 is the least fraction. So, Company B has better watches.

2. Sale of English and Hindi books in the years 1995, 1996, 1997 and 1998 are given below:

Years	1995	1996	1997	1998
English	350	400	450	620
Hindi	500	525	600	650

Draw a double bar graph and answer the following questions: (a) In which year was the difference in the sale of the two language books least?.

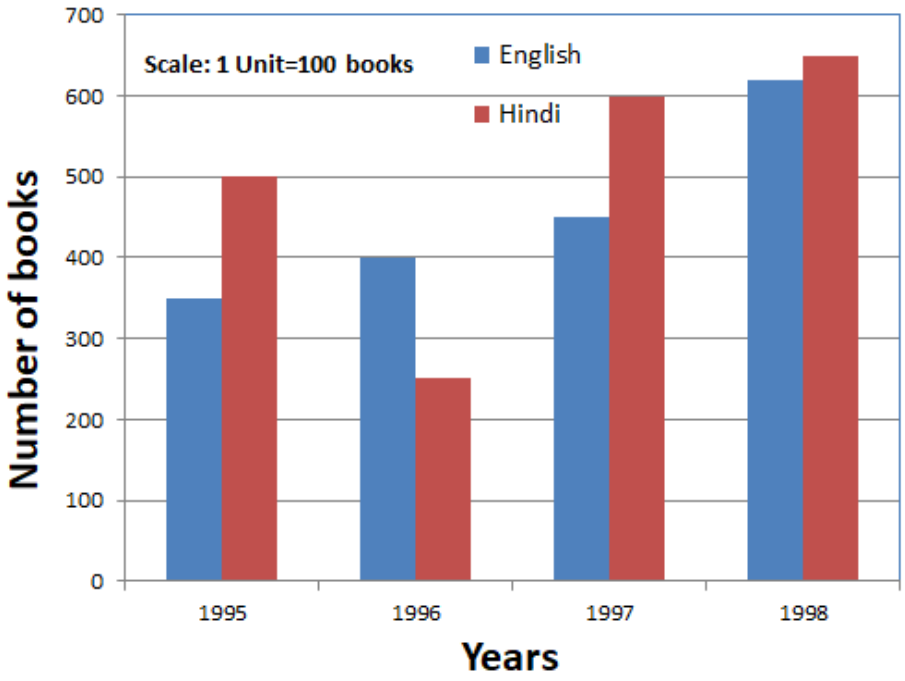
Sol:1998

(b) Can you say that the demand for English books rose faster? Justify

Sol: Yes, I can say that the demand for English books rose faster.

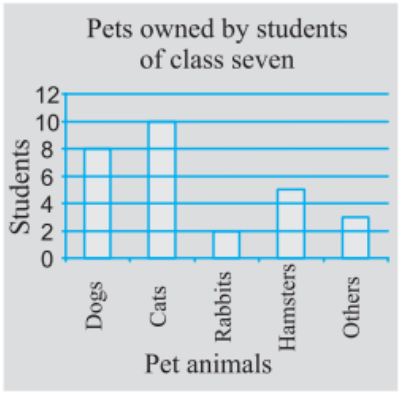
The demand of English books from 1995 to 1998 =620-350=270

The demand of Hindi books from 1995 to 1998 =650-500=150



EXERCISE 3.3

1. **Use the bar graph (Fig 3.3) to answer the following questions**



(a) Which is the most popular pet?

Sol: Cat

(b) How many students have dog as a pet?

Sol: 8 students

2.Read the bar graph (Fig 3.4) which shows the number of books sold by a bookstore during five consecutive years and answer the following questions:

(i) About how many books were sold in 1989? 1990? 1992?

Sol: Number of books sold in 1989=180

Number of books sold in 1990=475

Number of books sold in 1992=225

(ii) In which year were about 475 books sold? About 225

books sold?

Sol: In 1990, about 475 books sold . In 1992, about 225 books sold.

(iii) In which years were fewer than 250 books sold?

Sol: In 1989 and 1992 fewer than 250 books were sold.

(iv) Can you explain how you would estimate the number of books sold in 1989?

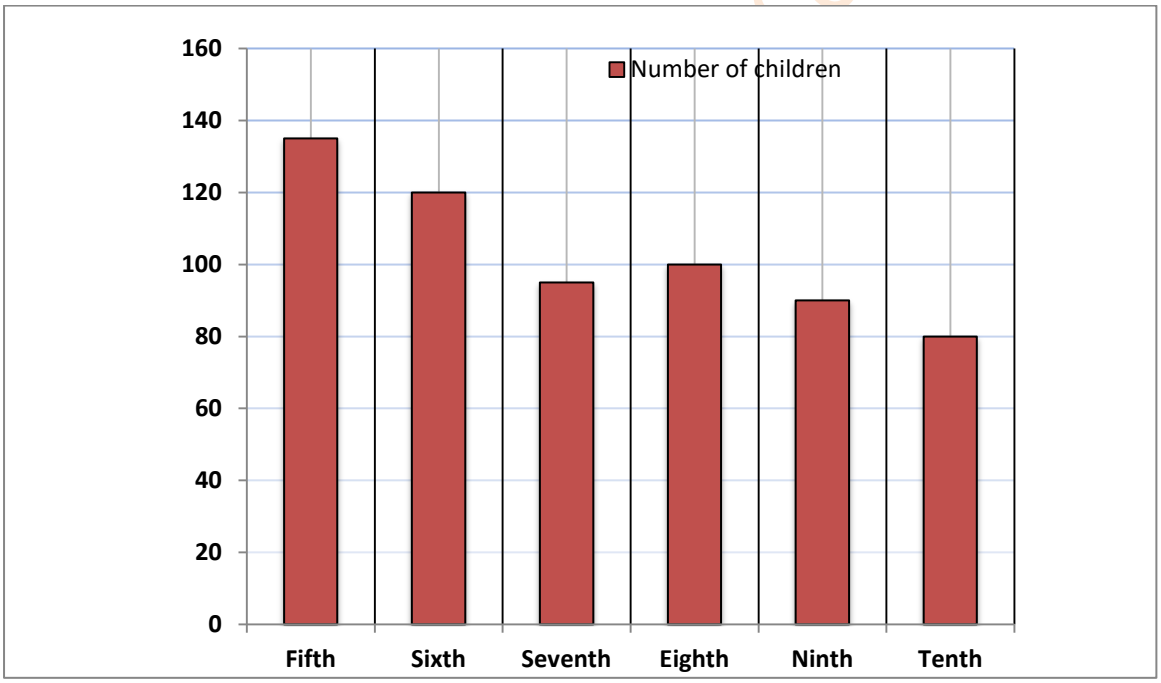
Sol: From the graph, I can estimate 180 books sold in 1989.

3. **Number of children in six different classes are given below. Represent the data on a bar graph**

Class	Fifth	Sixth	Seventh	Eighth	Ninth	Tenth
Number of Children	135	120	95	100	90	80

(a) How would you choose a scale? (b) Answer the following questions: (i) Which class has the maximum number of children? And the minimum? (ii) Find the ratio of students of class sixth to the students of class eight.

Sol:



(a) Scale: on Y-axis 1 unit=10 children

(b) (i) Fifth class has the maximum number of children and tenth class has the minimum number of children.

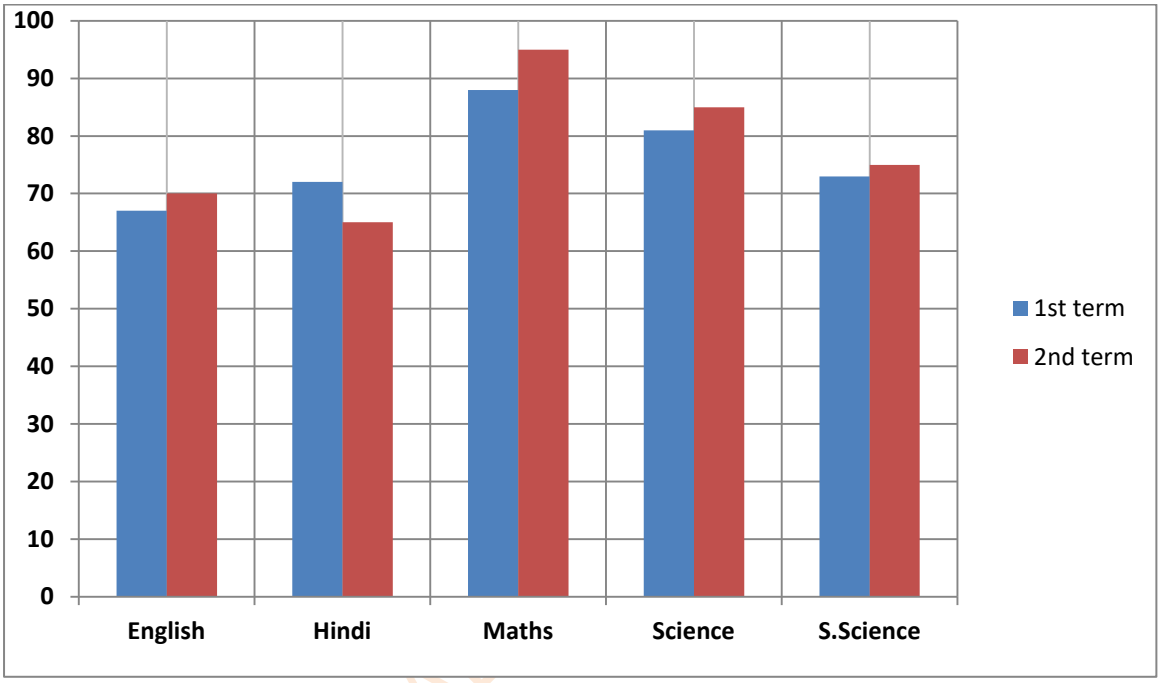
(ii) The ratio of students of class sixth to the students of class eighth=120:100=6:5

4. The performance of a student in 1st Term and 2nd Term is given. Draw a double bar graph choosing appropriate scale and answer the following:

Subject	English	Hindi	Maths	Science	S. Science
1st Term (M.M. 100)	67	72	88	81	73
2nd Term (M.M. 100)	70	65	95	85	75

(i) In which subject, has the child improved his performance the most? (ii) In which subject is the improvement the least? (iii) Has the performance gone down in any subject?

Sol:

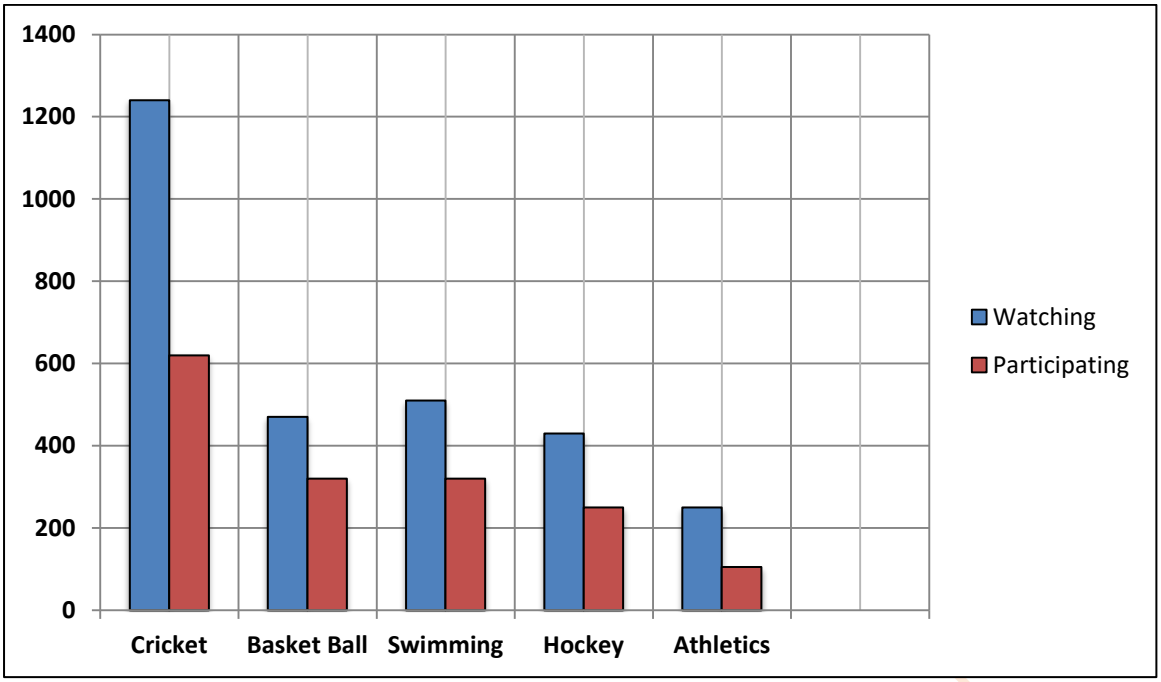


5. Consider this data collected from a survey of a colony.

Favourite Sport	Cricket	Basket Ball	Swimming	Hockey	Athletics
Watching	1240	470	510	430	250
Participating	620	320	320	250	105

(i) Draw a double bar graph choosing an appropriate scale. What do you infer from the bar graph? (ii) Which sport is most popular? (iii) Which is more preferred, watching or participating in sports?

Sol:



6. Take the data giving the minimum and the maximum temperature of various cities given in the beginning of this Chapter (Table 3.1). Plot a double bar graph using the data and answer the following:

(i) Which city has the largest difference in the minimum and maximum temperature on the given date? (ii) Which is the hottest city and which is the coldest city? (iii) Name two cities where maximum temperature of one was less than the minimum temperature of the other. (iv) Name the city which has the least difference between its minimum and the maximum temperature.

Sol:

1. A variable takes on different numerical values; its value is not fixed. Variables are denoted usually by letters of the alphabets, such as x, y, z, l, m, n, p, etc
2. The expressions are formed by performing operations like addition, subtraction, multiplication and division on the variables.

   Ex: $4x + 5, 3y - 10, -2z - 6, \ldots$

TRY THESE

The value of the expression (10y – 20) depends on the value of y. Verify this by giving five different values to y and finding for each y the value of (10 y – 20). From the different values of (10y – 20) you obtain, do you see a solution to 10y – 20 = 50? If there is no solution, try giving more values to y and find whether the condition 10y – 20 = 50 is met.

Sol:

Value of y	Value of 10y-20
$y = 0$	$10 \times 0 - 20 = 0 - 20 = -20$
$y = 1$	$10 \times 1 - 20 = 10 - 20 = -10$
$y = 2$	$10 \times 2 - 20 = 20 - 20 = 0$
$y = 3$	$10 \times 3 - 20 = 30 - 20 = 10$
$y = 4$	$10 \times 4 - 20 = 40 - 20 = 20$
$y = 5$	$10 \times 5 - 20 = 50 - 20 = 30$
$y = 6$	$10 \times 6 - 20 = 60 - 20 = 40$
$y = 7$	$10 \times 7 - 20 = 70 - 20 = 50$

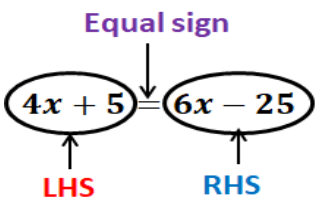
When we take y = 7, the condition 10y - 20 = 50 is met.

EQUATION

- In an equation there is always an equality sign.
- An equation is a condition on a variable. The condition is that two expressions should have equal value. Note that at least one of the two expressions must contain the variable
- The expression to the left of the equal sign is **LHS** and the left of the equal sign is **RHS**.
- An equation remains the same, when the expressions on the left and on the right are interchanged

Ex: In equation : $\boldsymbol{4x + 5 = 6x - 25}$

LHS $= 4x + 5$ and RHS $= 6x - 25$



Example 1: Write the following statements in the form of equations:

(i) The sum of three times x and 11 is 32.

Sol: $3x + 11 = 32$

(ii) If you subtract 5 from 6 times a number, you get 7.

Sol: Let the number=z

The equation is $6z - 5 = 7$

(iii) One fourth of m is 3 more than 7.

Sol: $\frac{m}{4} - 7 = 3$

(iv) One third of a number plus 5 is 8.

Sol: Let the number=n

The equation is $\frac{n}{3} + 5 = 8$

Example2: Convert the following equations in statement form:

(i) $x - 5 = 9$

Sol: Taking away 5 from x gives 9

(ii) $5p = 20$

Sol: Five times a number p is 20.

(iii) $3n + 7 = 1$

Sol: Add 7 to three times n to get 1.

(iv) $\frac{m}{5} - 2 = 6$

Sol: You get 6, when you subtract 2 from one-fifth of a number m

Example 3 :Consider the following situation: Raju's father's age is 5 years more than three times Raju's age. Raju's father is 44 years old. Set up an equation to find Raju's age.

Sol: Let Raju's age=y years.

Raju's father's age= $(3y + 5) years$

From problem Raju's fathers age= 44 years

$\therefore$ Required equation : $3y + 5 = 44$

Example 4: A shopkeeper sells mangoes in two types of boxes, one small and one large. A large box contains as many as 8 small boxes plus 4 loose mangoes. Set up an equation which gives the number of mangoes in each small box. The number of mangoes in a large box is given to be 100.

Sol: Let number of mangoes in small box=m

Number of mangoes in large box=$8m + 4$

Given the number of mangoes in large box=100

$\therefore$ Required equation: $8m + 4 = 100$

EXERCISE 4.1

1. Complete the last column of the table.

S. No	Equation	Value	Say, whether the Equation is Satisfied. (Yes/ No)	
(i)	$x + 3 = 0$	$x = 3$	LHS = x + 3 = 3 + 3 = 6 ≠ RHS	No
(ii)	$x + 3 = 0$	$x = 0$	LHS = x + 3 = 0 + 3 = 3 ≠ RHS	No
(iii)	$x + 3 = 0$	$x = -3$	LHS = x + 3 = −3 + 3 = 0 = RHS	Yes
(iv)	$x - 7 = 1$	$x = 7$	LHS = x − 7 = 7 − 7 = 0 ≠ RHS	No
(v)	$x - 7 = 1$	$x = 8$	LHS = x − 7 = 8 − 7 = 1 = RHS	Yes
(vi)	$5x = 25$	$x = 0$	LHS = $5x = 5 \times 0 = 0 \neq$ RHS	No
(vii)	$5x = 25$	$x = 5$	LHS = $5x = 5 \times 5 = 25 =$ RHS	Yes
(viii)	$5x = 25$	$x = -5$	LHS = $5x = 5 \times (-5) = -25 \neq$ RHS	No
(ix)	$\frac{m}{3} = 2$	$m = -6$	LHS = $\frac{m}{3} = \frac{-6}{3} = -2 \neq$ RHS	No
(x)	$\frac{m}{3} = 2$	$m = 0$	LHS = $\frac{m}{3} = \frac{0}{3} = 0 \neq$ RHS	No
(xi)	$\frac{m}{3} = 2$	$m = 6$	LHS = $\frac{m}{3} = \frac{6}{3} = 2 =$ RHS	Yes

2. Check whether the value given in the brackets is a solution to the given equation or not:

$(a)\ n + 5 = 19\ (n = 1)$

Sol: LHS = $n + 5 = 1 + 5 = 6$

RHS = 19

LHS ≠ RHS.

$So, n = 1$ *is not a solution of* $n + 5 = 19$

(b) $\boldsymbol{7n + 5 = 19\ (n = -2)}$

Sol: LHS $= 7n + 5 = 7 \times (-2) + 5 = -14 + 5 = -9$

RHS $= 19$

LHS $\neq$ RHS.

$So, n = -2$ *is not a solution of* $7n + 5 = 19$

(c) $\boldsymbol{7n + 5 = 19\ (n = 2)}$

Sol: LHS $= 7n + 5 = 7 \times (2) + 5 = 14 + 5 = 19$

RHS $= 19$

LHS $=$ RHS.

So, $n = 2$ is a solution of $7n + 5 = 19$

(d) $\boldsymbol{4p - 3 = 13\ (p = 1)}$

Sol: LHS $= 4p - 3 = 4 \times 1 + 5 = 4 + 5 = 9$

RHS $= 13$

LHS $\neq$ RHS.

So, $p = 1$ is not a solution of $4p - 3 = 13$

(e) $\boldsymbol{4p - 3 = 13\ (p = -4)}$

Sol: LHS $= 4p - 3 = 4 \times (-4) - 3 = -16 - 3 = -19$

RHS $= 13$

LHS $\neq$ RHS.

So, $p = -4$ is not a solution of $4p - 3 = 13$

(f) $\boldsymbol{4p - 3 = 13\ (p = 0)}$

Sol: LHS $= 4p - 3 = 4 \times 0 - 3 = 0 - 3 = -3$

RHS $= 13$

LHS $\neq$ RHS.

$So, p = 0$ is not a solution of $4p - 3 = 13$

3. Solve the following equations by trial and error method:

(i) $\boldsymbol{5p + 2 = 17}$

'p' value	$LHS = 5p + 2$	RHS=17	Is LHS=RHS
0	$5p + 2 = 5 \times 0 + 2 = 0 + 2 = 2$	17	No
1	$5p + 2 = 5 \times 1 + 2 = 5 + 2 = 7$	17	No
2	$5p + 2 = 5 \times 2 + 2 = 10 + 2 = 12$	17	No
3	$5p + 2 = 5 \times 3 + 2 = 15 + 2 = 17$	17	Yes

For p = 3, LHS = RHS.

So, p = 3 is the solution of the equation $5p + 2 = 17$.

$(ii)\mathbf{3m - 14 = 4}$

$'m'value.$	LHS $= 3m - 14$	RHS=4	Is LHS=RHS
2	$3m - 14 = 3 \times 2 - 14 = 6 - 14 = -8$	4	No
3	$3m - 14 = 3 \times 3 - 14 = 9 - 14 = -5$	4	No
4	$3m - 14 = 3 \times 4 - 14 = 12 - 14 = -2$	4	No
5	$3m - 14 = 3 \times 5 - 14 = 15 - 14 = 1$	4	No
6	$3m - 14 = 3 \times 6 - 14 = 18 - 14 = 4$	4	Yes

For m=6, LHS=RHS.

So, m=6 is the solution of the equation $3m - 14 = 4$

4. Write equations for the following statements:

(i) The sum of numbers x and 4 is 9.

Sol: $x + 4 = 9$

(ii) 2 subtracted from y is 8.

Sol: $y - 2 = 8$

(iii) Ten times a is 70.

Sol: $10a = 70$

(iv) The number b divided by 5 gives 6.

Sol: $\frac{b}{5} = 6$

(v) Three-fourth of t is 15.

$Sol: \frac{3}{4}t = 15\ or\ \frac{3t}{4} = 15$

(vi) Seven times m plus 7 gets you 77.

Sol: $7m + 7 = 77$

(vii) One-fourth of a number x minus 4 gives 4.

Sol: $\frac{1}{4}x - 4 = 4\ or\ \frac{x}{4} - 4 = 4$

(viii) If you take away 6 from 6 times y, you get 60.

Sol: $6y - 6 = 60$

(ix) If you add 3 to one-third of z, you get 30.

Sol: $\frac{1}{3}z + 3 = 30\ \ or\ \frac{z}{3} + 3 = 30$

5. **Write the following equations in statement forms:**

(i) $p + 4 = 15$

Sol: The sum of p and 4 is 15.

(ii) $m - 7 = 3$

Sol: 7subtracted from m is 3.

(iii) $\mathbf{2m = 7}$

Sol: Twice of a number m is 7.

(iv) $\frac{\boldsymbol{m}}{\mathbf{5}} = \mathbf{3}$

Sol: One $-$ fifth of a number m is 3.

(v) $\frac{\mathbf{3m}}{\mathbf{5}} = \mathbf{6}$

Sol: Three $-$ fifth of a number m *is* 6.

(vi) $\mathbf{3p + 4 = 25}$

Sol: Three times of a number p when added to 4 gives 25

(vii) $\mathbf{4p - 2 = 18}$

Sol: 2 subtracted from four times a number p is 18.

(viii) $\frac{\boldsymbol{p}}{\mathbf{2}} + \mathbf{2} = \mathbf{8}$

Sol: Add 2 to half of a number p to get 8.

6. Set up an equation in the following cases:

(i) **Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. (Take m to be the number of Parmit's marbles.)**

Sol: Let number of marbles Parmit has $= m$

Number of marbles Irfan has $= 5m + 7$

But Ifran has 37 marbles

Required equation: $5m + 7 = 37$

(ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. (Take Laxmi's age to be y years.)

Sol: Let Lakshmi's age $= y$ years

Lakshmi's father age $= 3y + 4$

But Lakshmi's father age=49 years

Required equation: $3y + 4 = 49$

(iii) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. (Take the lowest score to be l.)

Sol: Let the lowest score $= l$

The highest score $= 2l + 7$

But the highest score=87

Required equation: $2l + 7 = 87$

(iv) In an isosceles triangle, the vertex angle is twice either base angle. (Let the base angle be b in degrees. Remember that the sum of angles of a triangle is 180 degrees).

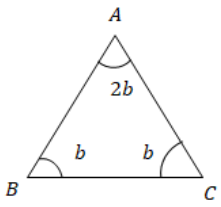
Sol: Let base angle of the triangle $= b$ degrees.

The vertex angle $= 2b$

The sum of angles of a triangle is 180 degrees

$b + b + 2b = 180^0$

Required equation: $4b = 180^0$



Solving an Equation

1. If we **add** or **subtract** the same number from both sides of an equation, it still holds.
2. If we **multiply** or **divide** both sides of the equation by the same non-zero number, it still holds.
3. If we fail to do the same mathematical operation with same number on both sides of an equation, the equality may not hold.

Example 5: $\boldsymbol{Solve\ 3n + 7 = 25}$

Sol: Given equation $3n + 7 = 25$

Subtracting 7 from both sides

$3n + 7 - 7 = 25 - 7$

$3n = 18$

Divide both sides by 3,

$\frac{3n}{3} = \frac{18}{3}$

$n = 6$

$n = 6\ is\ the\ solution\ of\ the\ equation\ 3n + 7 = 25$

(b) Solve $2p - 1 = 23$

Sol: Given equation : $2p - 1 = 23$

Add 1 to both the sides.

$2p - 1 + 1 = 23 + 1$

$2p = 22$

Divide both sides by 2,

$\frac{2p}{2} = \frac{22}{2}$

$p = 11$

$p = 11$ is the solution of the equation $2p - 1 = 23$.

EXERCISE 4.2

1. Give first the step you will use to separate the variable and then solve the equation:

$(a)\ x - 1 = 0$

Sol: $x - 1 = 0$

Add '1' on both sides.

$x - 1 + 1 = 0 + 1$

$x = 1$

Solution : $x = 1$

$(b) x + 1 = 0$

Sol: $x + 1 = 0$

Subtract '1' on both sides.

$x + 1 - 1 = 0 - 1$

$x = -1$

Solution : $x = -1$

(c) $x - 1 = 5$

Sol: $x - 1 = 5$

Add '1' on both sides.

$x - 1 + 1 = 5 + 1$

$x = 6$

Solution : $x = 6$

(d) $x + 6 = 2$

Sol: $x + 6 = 2$

Subtract '6' from both sides.

$x + 6 - 6 = 2 - 6$

$x = -4$

(e) $y - 4 = -7$

Sol: $y - 4 = -7$

Add '4' on both sides

$y - 4 + 4 = -7 + 4$

$y = -3$

(f) $y - 4 = 4$

Sol: $y - 4 = 4$

Add '4' on both sides

$y - 4 + 4 = 4 + 4$

$y = 8$

(g) $y + 4 = 4$

Sol: $y + 4 = 4$

Subtract '4' from both sides.

$y + 4 - 4 = 4 - 4$

$y = 0$

(h) $y + 4 = -4$

Sol: $y + 4 = -4$

Subtract '4' from both sides.

$y + 4 - 4 = -4 - 4$

$y = -8$

2. Give first the step you will use to separate the variable and then solve the equation:

$(a) 3l = 42$

$Sol: 3l = 42$

Divide both sides by '3'.

$\frac{3l}{3} = \frac{42}{3}$

$l = 14$

$(b) \frac{b}{2} = 6$

Sol: $\frac{b}{2} = 6$

Multiply both sides with '2'.

$\frac{b}{2} \times 2 = 6 \times 2$

$b = 12$

$(c) \frac{p}{7} = 4$

Sol: $\frac{p}{7} = 4$

Multiply both sides with '7'

$\frac{p}{7} \times 7 = 4 \times 7$

$p = 28$

$(d) 4x = 25$

Sol: $4x = 25$

Divide both sides by '4'

$\frac{4x}{4} = \frac{25}{4}$

$x = \frac{25}{4}$

$(e) 8y = 36$

Sol: $8y = 36$

Divide both sides by '8'

$\frac{8y}{8} = \frac{36}{8}$

$y = \frac{9}{2}$

(f) $\frac{z}{3} = \frac{5}{4}$

Sol: $\frac{z}{3} = \frac{5}{4}$

Multiply both sides with '3'

$\frac{z}{3} \times 3 = \frac{5}{4} \times 3$

$z = \frac{15}{4}$

(g) $\frac{a}{5} = \frac{7}{15}$

Sol: $\frac{a}{5} = \frac{7}{15}$

Multiply both sides with '5'

$\frac{a}{5} \times 5 = \frac{7}{15} \times 5$

$a = \frac{7}{3}$

(h) $20t = -10$

Sol: $20t = -10$

Divide both sides by '20'

$\frac{20t}{20} = \frac{-10}{20}$

$t = -\frac{1}{2}$

3. Give the steps you will use to separate the variable and then solve the equation:

(a) $3n - 2 = 46$

Sol: $3n - 2 = 46$

Add '2' on both sides.

$3n - 2 + 2 = 46 + 2$

$3n = 48$

Divide both sides by '3'

$\frac{3n}{3} = \frac{48}{3}$

$n = 16$

(b) $5m + 7 = 17$

Sol: $5m + 7 = 17$

Subtract '7' from both sides.

$5m + 7 - 7 = 17 - 7$

$5m = 10$

Divide both sides by '5'

$\frac{5m}{5} = \frac{10}{5}$

$m = 2$

(c) $\frac{20p}{3} = 40$

Sol: $\frac{20p}{3} = 40$

Multiply both sides with '3'

$\frac{20p}{3} \times 3 = 40 \times 3$

$20p = 120$

Divide both sides by '20'

$\frac{20p}{20} = \frac{120}{20}$

$p = 6$

(d) $\frac{3p}{10} = 6$

Sol: $\frac{3p}{10} = 6$

Multiply both sides with '10'

$\frac{3p}{10} \times 10 = 6 \times 10$

$3p = 60$

Divide both sides by '3'

$\frac{3p}{3} = \frac{60}{3}$

$p = 20$

4. Solve the following equations:

$(a)\mathbf{10p = 100}$

Sol: $10p = 100$

Divide both sides by '10'

$\frac{10p}{10} = \frac{100}{10}$

$p = 10$

$(b)\mathbf{10p + 10 = 100}$

Sol: $10p + 10 = 100$

Subtract '10' from both sides.

$10p + 10 - 10 = 100 - 10$

$10p = 90$

Divide both sides by '10'

$\frac{10p}{10} = \frac{90}{10}$

$p = 9$

$(c)\mathbf{\frac{p}{4} = 5}$

Sol: $\frac{p}{4} = 5$

Multiply both sides with '4'.

$\frac{p}{4} \times 4 = 5 \times 4$

$p = 20$

$(d)\mathbf{\frac{-p}{3} = 5}$

Sol: $\frac{-p}{3} = 5$

Multiplied both sides with '-3'

$\frac{-p}{3} \times (-3) = 5 \times (-3)$

$P = -15$

$(e)\mathbf{\frac{3p}{4} = 6}$

Sol: $\frac{3p}{4} = 6$

Multiply both sides with '4'

$\frac{3p}{4} \times 4 = 6 \times 4$

$3p = 24$

Divide both sides by '3'

$\frac{3p}{3} = \frac{24}{3}$

$p = 8$

$(f)\mathbf{3s = -9}$

Sol: $3s = -9$

Divide both sides by '3'

$\frac{3s}{3} = \frac{-9}{3}$

$s = -3$

$(g)\mathbf{3s + 12 = 0}$

Sol: $3s + 12 = 0$

Subtract '12' from both sides.

$3s + 12 - 12 = 0 - 12$

$3s = -12$

Divide both sides by '3'

$\frac{3s}{3} = \frac{-12}{3}$

$s = -4$

$(h)\mathbf{3s = 0}$

Sol: $3s = 0$

Divide both sides by '3'

$\frac{3s}{3} = \frac{0}{3}$

$s = 0$

$(i)\mathbf{2q = 6}$

Sol: $2q = 6$

Divide both sides by '2'

$\frac{2q}{2} = \frac{6}{3}$

$q = 2$

$(j)\mathbf{2q - 6 = 0}$

Sol: $2q - 6 = 0$

Add '6' on both sides.

$2q - 6 + 6 = 0 + 6$

$2q = 6$

Divide both sides by '2'

$\frac{2q}{2} = \frac{6}{3}$

$q = 2$

$(k) 2q + 6 = 0$

Sol: $2q + 6 = 0$

Subtract '6' from both sides.

$2q + 6 - 6 = 0 - 6$

$2q = -6$

Divide both sides by '2'

$\frac{2q}{2} = \frac{-6}{3}$

$q = -2$

$(l) 2q + 6 = 12$

Subtract '6' from both sides.

$2q + 6 - 6 = 12 - 6$

$2q = 6$

Divide both sides by '2'

$\frac{2q}{2} = \frac{6}{3}$

$q = 2$

Method of transposition:

Transposing a number (i.e., changing the side of the number) is the same as adding or subtracting multiply or dividing the number from both sides

Thus, in transposing terms from LHS to RHS or RHS to LHS

'+ quantity' becomes '– quantity'	'– quantity' becomes '+ quantity'
'× quantity' becomes '÷ quantity'	'÷ quantity' becomes '× quantity'

Exp 6 :Solve: 12p – 5 = 25 .

Sol: Given equation: $12p - 5 = 25$.

$12p = 25 + 5$ (transposing -5 *to RHS*)

$12p = 30$

$\frac{12p}{12} = \frac{30}{12}$ (Dicvide both sides by 12)

$p = \frac{5}{2}$

Check: Putting $p = \frac{5}{2}$

$\text{LHS} = 12p - 5 = 12 \times \frac{5}{2} - 5$

$= 6 \times 5 - 5$

$= 30 - 5$

$= 25 = \text{RHS}$

***Example* 7: *Solve* $(a) 4(m + 3) = 18$**

Sol: $4(m + 3) = 18$

Divide both sides by '4'.

$\frac{4(m+3)}{4} = \frac{18}{4}$

$m + 3 = \frac{9}{2}$

$m = \frac{9}{2} - 3$ (transposing 3 to RHS)

$m = \frac{9-6}{2} = \frac{3}{2}$

Check: $Put\ m = \frac{3}{2}$

LHS $= 4(m + 3) =$

$4\left(\frac{3}{2} + 3\right) = 4\left(\frac{3+6}{2}\right)$

$= 4 \times \frac{9}{2} = 2 \times 9$

$= 18 =$ RHS

(b)– $2(x\ +\ 3) =\ 8$

Sol: $-\ 2(x\ +\ 3) =\ 8$

Divide both sides by '-2'.

$\frac{-\ 2 \times (x\ +\ 3)}{-2} = \frac{8}{-2}$

$x + 3 =\ -4$

$x = -4 - 3$ (transposing 3 to RHS)

$x = -7$

Check: $Put\ x = -7$

LHS $=-\ 2(x\ +\ 3)$

$=-\ 2(-7\ +\ 3)$

$= -2 \times (-4)$

$=\ 8$

$=$ RHS

Exmple 8:The sum of three times a number and 11 is 32. Find the number.

Sol: Let the number $= x$

From problem: $3x + 11 = 32$

$3x = 32 - 11$(transposing 11 to RHS)

$3x = 21$

Divide both sides by '3'

$\frac{3x}{3} = \frac{21}{3}$

$x = 7$

The required number is 7.

Example 9: Find a number, such that one-fourth of the number is 3 more than 7.

Sol: Let the number $= y$

$\frac{y}{4} - 7 = 3$

$\frac{y}{4} = 3 + 7$

$\frac{y}{4} = 10$

$\frac{y}{4} \times 4 = 10 \times 4$

$y = 40$

The required number is 40.

Example 10 :Raju's father's age is 5 years more than three times Raju's age. Find Raju's age, if his father is 44 years old.

Sol: Let Raju's age $= x$ years

Raju's father's age $= (3x + 5)$ years.

Given Raju's father's age=44 years.

$3x + 5 = 44$

$3x = 44 - 5$

$3x = 39$

$\frac{3x}{3} = \frac{39}{3}$

$x = 13$

Raju's age=13 years.

Try These

There are two types of boxes containing mangoes. Each box of the larger type contains 4 more mangoes than the number of mangoes contained in 8 boxes of the smaller type. Each larger box contains 100 mangoes. Find the number of mangoes contained in the smaller box?

Sol: Let the number of mangoes in smaller box=x

Number of mangoes in larger box $= 8x + 4$

But each larger box contains 100 mangoes.

$$8x + 4 = 100$$

$$8x = 100 - 4$$

$$8x = 96$$

$$\frac{8x}{8} = \frac{96}{8}$$

$$x = 12$$

$\therefore$ The number of mangoes in smaller box $= 12$.

EXERCISE 4.3

1. **Set up equations and solve them to find the unknown numbers in the following cases:**

(a) Add 4 to eight times a number; you get 60.

Sol: Let the number $= x$

$$8x + 4 = 60$$

$$8x = 60 - 4$$

$$8x = 56$$

$$\frac{8x}{8} = \frac{56}{8}$$

$$x = 7$$

(b) One-fifth of a number minus 4 gives 3.

Sol: Let the number $= y$

$$\frac{y}{5} - 4 = 3$$

$$\frac{y}{5} = 3 + 4$$

$$\frac{y}{5} = 7$$

$$\frac{y}{5} \times 5 = 7 \times 5$$

$$y = 35$$

(c) If I take three-fourths of a number and add 3 to it, I get 21.

Sol: Let the number $= n$

$$\frac{3n}{4} + 3 = 21$$

$$\frac{3n}{4} = 21 - 3$$

$$\frac{3n}{4} = 18$$

$$\frac{3n}{4} = 18$$

$$\frac{3n}{4} \times \frac{4}{3} = 18 \times \frac{4}{3}$$

$$n = 6 \times 4 = 24$$

(d) When I subtracted 11 from twice a number, the result was 15.

Sol: Let the number $= x$

$$2x - 11 = 15$$

$$2x = 15 + 11$$

$$2x = 26$$

$$\frac{2x}{2} = \frac{26}{2}$$

$$x = 13$$

(e) Munna subtracts thrice the number of notebooks he has from 50, he finds the result to be 8.

Sol: Let the number of note books $= y$

$$50 - 3y = 8$$

$$-3y = 8 - 50$$

$$-3y = -42$$

$$\frac{-3y}{-3} = \frac{-42}{-3}$$

$$y = 14$$

(f) Ibenhal thinks of a number. If she adds 19 to it and divides the sum by 5, she will get 8.

Sol: Let the number $= n$

$$\frac{n + 19}{5} = 8$$

$$n + 19 = 5 \times 8$$

$$n + 19 = 40$$

$$n = 40 - 19 = 21$$

(g) Anwar thinks of a number. If he takes away 7 from $\frac{5}{2}$ of the number, the result is 23.

Sol: Let the number $= x$

$$\frac{5x}{2} - 7 = 23$$

$$\frac{5x}{2} = 23 + 7$$

$$\frac{5x}{2} = 30$$

$$\frac{5x}{2} \times \frac{2}{5} = 30 \times \frac{2}{5}$$

$$x = 6 \times 2$$

$$x = 12$$

2. **Solve the following:**

(a) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. What is the lowest score?

Sol: Let the lowest score $= x$

Highest marks $= 2x + 7$

Given the highest score=87

$$2x + 7 = 87$$

$$2x = 87 - 7$$

$$2x = 80$$

$$\frac{2x}{2} = \frac{80}{2}$$

$$x = 40$$

$\therefore$ The lowest score=40.

(b) In an isosceles triangle, the base angles are equal. The vertex angle is 40°. What are the base angles of the triangle? (Remember, the sum of three angles of a triangle is 180°).

Sol: Let the base angles $= b, b$

The vertex angle= 40°

The sum of three angles of a triangle is 180°

$$b + b + 40^0 = 180^0$$

$$2b + 40^0 = 180^0$$

$$2b = 180^0 - 40^0$$

$$2b = 140^0$$

$$\frac{2b}{2} = \frac{140^0}{2}$$

$$b = 70^0$$

$\therefore$ The base angles are $70^0, 70^0$.

(c) Sachin scored twice as many runs as Rahul. Together, their runs fell two short of a double century. How many runs did each one score?

Sol: Let Rahul's score $= x$

Sachin's score $= 2x$

From problem: $x + 2x = 200 - 2$

$3x = 198$

$\frac{3x}{3} = \frac{198}{3}$

$x = 66$

Rahul's score $= 66$ runs

Sachin's score $= 2x = 132$ runs

3. **Solve the following:**

(i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. How many marbles does Parmit have?

Sol: Let the number of marbles permit has$=x$

Number of marbles Irfan has$=5x + 7$

According to problem Irfan has 37 marbles.

$5x + 7 = 37$

$5x = 37 - 7$

$5x = 30$

$\frac{5x}{5} = \frac{30}{5}$

$x = 6$

$\therefore$ Parmit has 6 marbles.

(ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. What is Laxmi's age?

Sol: Let Laxmi's age$=x\ years$

From problem:$3x + 4 = 49$

$3x = 49 - 4$

$3x = 45$

$\frac{3x}{3} = \frac{45}{3}$

$x = 15$

$\therefore$ Laxmi's age$=15\ years$

(iii) People of Sundargram planted trees in the village garden. Some of the trees were fruit trees. The number of non-fruit trees were two more than three times the number of fruit trees. What was the number of fruit trees planted if the number of non-fruit trees planted was 77?

Sol: Let the number of fruit trees planted= x

From problem: $3x + 2 = 77$

$3x = 77 - 2$

$3x = 75$

$\frac{3x}{3} = \frac{75}{3}$

$x = 25$

$\therefore$ The number of fruit trees planted=25

4. Solve the following riddle:

I am a number, Tell my identity!

Take me seven times over, And add a fifty!

To reach a triple century, You still need forty!

Sol: Let the number=x

From the problem: $7x + 50 + 40 = 300$

$7x + 90 = 300$

$7x = 300 - 90$

$7x = 210$

$\frac{7x}{7} = \frac{210}{7}$

$x = 30$

$\therefore$ The required number $= 30$

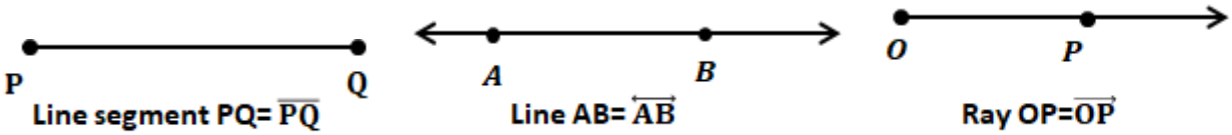
CHAPTER 5

VII-MATHEMATICS-NCERT-2023-24

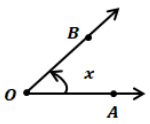
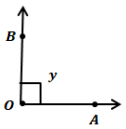
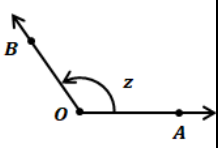
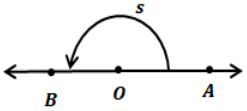
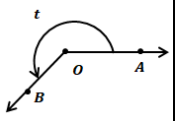
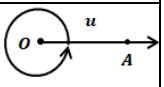
5. Lines and Angles (Notes)

PREPARED BY: BALABHADRA SURESH

1. A line segment has two end points.
2. If we extend the two end points of a line segment in either direction endlessly, we get a line.
3. A line has no end points.
4. A ray has one end point (namely its starting point).



5. An angle is made up of two rays starting from a common starting/initial point
6. **Types of angles**:

Name	Acute angle	Right angle	Obtuse angle	Straight angle	Reflex angle	Complete angle
Measure	$0^0 < x < 90^0$	$y = 90^0$	$90^0 < z < 180^0$	$s = 180^0$	$180^0 < t < 360^0$	$u = 360^0$
Illustration						

7. **Complementary Angles**: The sum of the measures of two angles is 90°, the angles are called complementary angles.

   Ex: $50^0, 40^0$; $25^0, 65^0$
8. The complement of $x^0 = 90^0 - x^0$

THINK, DISCUSS AND WRITE

1. Can two acute angles be complement to each other?

Sol: Yes.

2. Can two obtuse angles be complement to each other?

Sol: No, Sum of two obtuse angles is greater than 90^0.

3. Can two right angles be complement to each other?

Sol: No, sum of two right angles is 180^0 .

TRY THESE

1. Which pairs of following angles are complementary?

Sol: (i) $70^0+20^0=90^0$

70^0,20^0 are complementary angles.

(ii) $75^0+25^0=100^0$

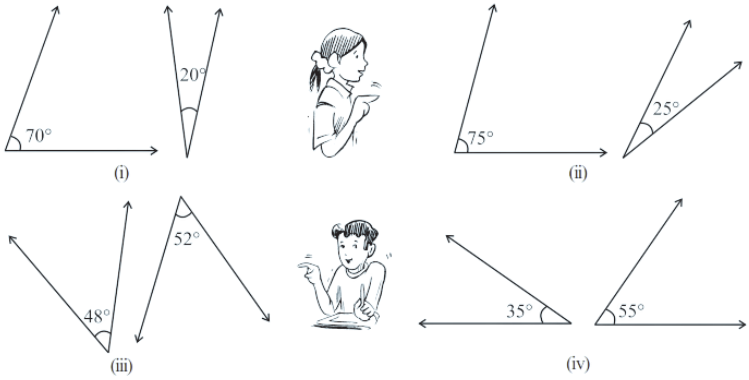
$75^0,25^0$ are not complementary angles.

(iii) $48^0+52^0=100^0$

$48^0,52^0$ are not complementary angles.

(iv) $35^0+55^0=90^0$

$35^0,55^0$ are complementary angles.



2. What is the measure of the complement of each of the following angles?

(i) 45º (ii) 65º (iii) 41º (iv) 54º

Sol: (i)The complementary angle of $45^0 = 90^0 - 45^0 = 45^0$

(ii)The complementary angle of $65^0 = 90^0 - 65^0 = 25^0$

(iii)The complementary angle of $41^0 = 90^0 - 41^0 = 49^0$

(iv)The complementary angle of $54^0 = 90^0 - 54^0 = 36^0$

3. **The difference in the measures of two complementary angles is 12^0 . Find the measures of the angles.**

Sol: Let the complementary angles are x and $x + 12^0$

Sum of complementary angles $= 90^0$

$x + x + 12^0 = 90^0$

$2x + 12^0 = 90^0$

$2x = 90^0 - 12^0$

$2x = 78^0$

$\frac{2x}{2} = \frac{78^0}{2}$

$x = 39^0$

$x + 12^0 = 39^0 + 12^0 = 51^0$

$\therefore$ The complementary angles are 39^0 and 51^0

Supplementary Angles: The sum of the measures of two angles is 180°, the angles are called supplementary angles.

The supplement of $x^0 = 180^0 - x^0$

THINK, DISCUSS AND WRITE

1. Can two obtuse angles be supplementary?

Sol: No, sum of two obtuse angles is greater than 180^0.

2. Can two acute angles be supplementary?

Sol: No, sum of two acute angles is less than 180^0.

3. Can two right angles be supplementary?

Sol: Yes, sum of two right angles is 180^0.

Try These

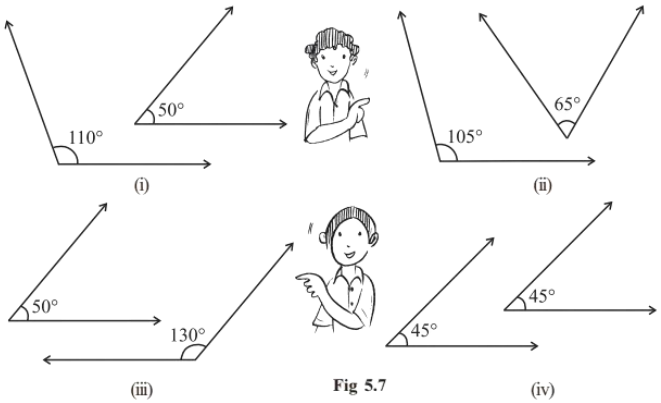


Fig 5.7

1. .Find the pairs of supplementary angles.

Sol: (i) $110^0 + 50^0 = 160^0$

$110^0$and 50^0 are not supplementary angles.

(ii) $105^0 + 65^0 = 170^0$

$105^0$and 65^0 are not supplementary angles.

(iii) $50^0 + 130^0 = 180^0$

$50^0$and 130^0 are supplementary angles.

(iv) $45^0 + 45^0 = 90^0$

$45^0$and 45^0 are not supplementary angles.

2. What will be the measure of the supplement of each one of the following angles?

The supplement of $x^0 = 180^0 - x^0$

(i) The supplement of $100^0 = 180^0 - 100^0 = 80^0$

(ii) The supplement of $90^0 = 180^0 - 90^0 = 90^0$

(iii) The supplement of $55^0 = 180^0 - 55^0 = 125^0$

(iv) The supplement of $125^0 = 180^0 - 125^0 = 55^0$

3. **Among two supplementary angles the measure of the larger angle is 44^0 more than the measure of the smaller. Find their measures.**

Sol: Let smaller angle $= x$

The larger angle $= x + 44^0$

Sum of two supplementary angles $= 180^0$

$$x + x + 44^0 = 180^0$$

$$2x + 44^0 = 180^0$$

$$2x = 180^0 - 44^0$$

$$2x = 136^0$$

$$\frac{2x}{2} = \frac{136^0}{2}$$

$$x = 68^0$$

Hence, smaller angle$=68^0$

Larger angle$=68^0 + 44^0 = 112^0$

EXERCISE 5.1

1. **Find the complement of each of the following angles:**

(i) Complement of $20^0 = 90^0 - 20^0 = 70^0$

(ii) Complement of $63^0 = 90^0 - 63^0 = 27^0$

(iii) Complement of $57^0 = 90^0 - 57^0 = 33^0$

2. **Find the supplement of each of the following angles:**

(i) Supplement of $105^0 = 180^0 - 105^0 = 75^0$

(ii) Supplement of $87^0 = 180^0 - 87^0 = 93^0$

(iii) Supplement of $154^0 = 180^0 - 154^0 = 26^0$

3. **Identify which of the following pairs of angles are complementary and which are supplementary.**

(i) 65º, 115º (ii) 63º, 27º (iii) 112º, 68º (iv) 130º, 50º (v) 45º, 45º (vi) 80º, 10º

Sol: (i) $65^0+115^0=180^0$

$65^0,115^0$ are supplementary angles.

(ii) $63^0+27^0=90^0$

$63^0,27^0$ are complementary angles.

(iii) $112^0+68^0=180^0$

$112^0,68^0$ are supplementary angles.

(iv) $130^0+50^0=180^0$

$35^0,55^0$ are supplementary angles.

(v) $45^0+45^0=90^0$

$45^0,45^0$ are complementary angles.

(vi) $80^0+10^0=90^0$

$80^0,10^0$ are complementary angles.

4. **Find the angle which is equal to its complement.**

Sol: Let the complementary angles are x *and* x

$x + x = 90^0$

$2x = 90^0$

$\frac{2x}{2} = \frac{90^0}{2}$

$x = 45^0$

5. **Find the angle which is equal to its supplement.**

Sol: Let the supplementary angles are x *and* x

$x + x = 180^0$

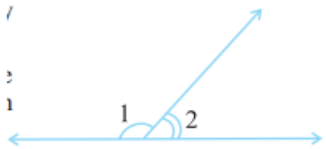
$2x = 180^0$

$\frac{2x}{2} = \frac{180^0}{2}$

$x = 90^0$

6. **In the given figure, ∠1 and ∠2 are supplementary angles. If ∠1 is decreased, what changes should take place in ∠2 so that both the angles still remain supplementary**

Sol: ∠2 will increase with the same measure as the decrease in ∠1.



7. **Can two angles be supplementary if both of them are:**

(i) acute? No (ii) obtuse? No (iii) right? Yes

8. **An angle is greater than 45º. Is its complementary angle greater than 45º or equal to 45º or less than 45º?**

Sol: Let the angle=50^0

Its complementary angle=$90^0-50^0=40^0$ is less than 45^0

If an angle is greater than 45º its complementary angle less than 45^0.

9. **Fill in the blanks:**

(i) If two angles are complementary, then the sum of their measures is **90^0 .**

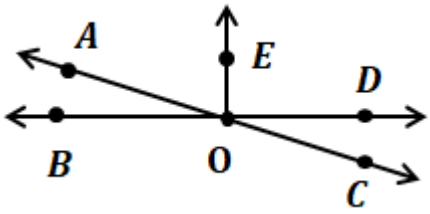
(ii) If two angles are supplementary, then the sum of their measures is **180^0.**

(iii) If two adjacent angles are supplementary, they form a **linear pair.**

10. **In the adjoining figure, name the following pairs of angles.**

(i) **Obtuse vertically opposite angles.**

Sol: ∠AOD, ∠BOC



(ii) **Adjacent complementary angles.**

Sol: ∠EOA, ∠AOB

(iii) Equal supplementary angles.

Sol: ∠EOB, ∠EOD

(iv) Unequal supplementary angles.

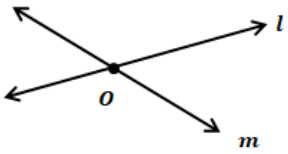
Sol: ∠EOA, ∠EOC

(v) Adjacent angles that do not form a linear pair.

Sol: ∠AOB, ∠AOE; ∠AOE, ∠EOD; ∠EOD, ∠COD

Intersecting Lines

1. If two lines have one common point, they are called intersecting lines.
2. Two lines l and m intersect if they have a point in common.
3. This common point O is their point of intersection.



Try These

1. **Find examples from your surroundings where lines intersect at right angles .**

Sol: (i) Adjacent edges of table.

(ii) Adjacent walls of a house.

(iii) Edges of black board.

2. **Find the measures of the angles made by the intersecting lines at the vertices of an equilateral triangle.**

Sol: 60^0

3. **Draw any rectangle and find the measures of angles at the four vertices made by the intersecting lines.**

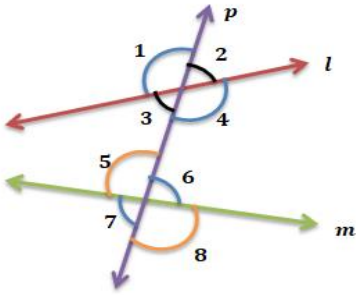
Sol: Each angle=90^0.

4. **If two lines intersect, do they always intersect at right angles?**

Sol: No, two intersecting lines do not always intersect at right angles.

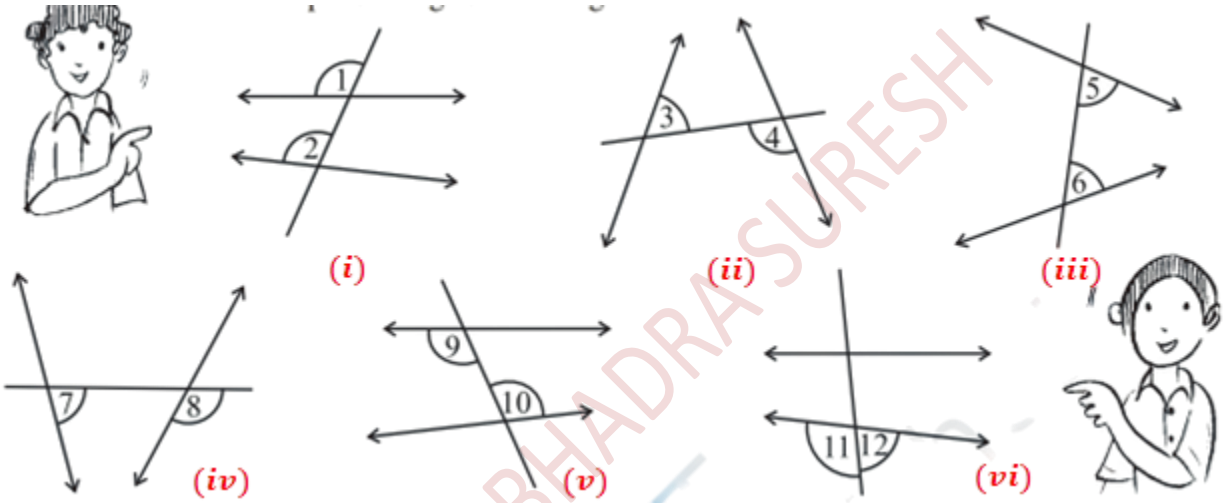
Transversal : A line that intersects two or more lines at distinct points is called a transversal.

Angles made by a Transversal:

Interior angles	∠3, ∠4, ∠5, ∠6	
Exterior angles	∠1, ∠2, ∠7, ∠8	
Pairs of Corresponding angles	∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8	
Pairs of Alternate interior angles	∠3 and ∠6, ∠4 and ∠5	
Pairs of interior angles on the same side of the transversal	∠3 and ∠5, ∠4 and ∠6	

Try These

Name the pairs of angles in each figure:



Sol: (i) ∠1 *and* ∠2 are pair of corresponding angles.

(ii) ∠3 *and* ∠4 are pair of alternate interior angles.

(iii) ∠5 *and* ∠6 are pair of interior angles on the same side of the transversal (Co-interior angles) (iv) ∠7 *and* ∠8 are pair of corresponding angles.

(v) ∠9 *and* ∠10 are pair of alternate interior angles.

(vi) ∠11 *and* ∠12 are pair of exterior angles.

Transversal of Parallel Lines:

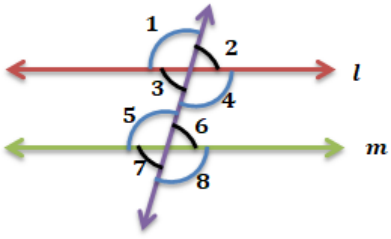
If f two parallel lines are cut by a transversal then

(i) Each pair of corresponding angles are equal in measure.

$\angle 1 = \angle 5;\ \angle 2 = \angle 6;\ \angle 3 = \angle 7;\ \angle 4 = \angle 8$

(ii) Each pair of alternate interior angles are equal.

$\angle 3 = \angle 6;\ \angle 4 = \angle 5$



(iii) Each pair of interior angles on the same side of the transversal are supplementary.

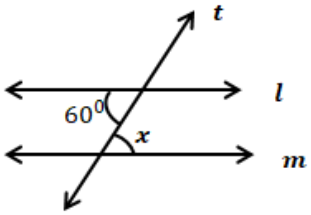
$\angle 3 + \angle 5 = 180^0$; $\angle 4 + \angle 6 = 180^0$

(iv) each pair of exterior angles on the same side of the transversal are supplementary

$\angle 1 + \angle 7 = 180^0$; $\angle 2 + \angle 8 = 180^0$

TRY THESE

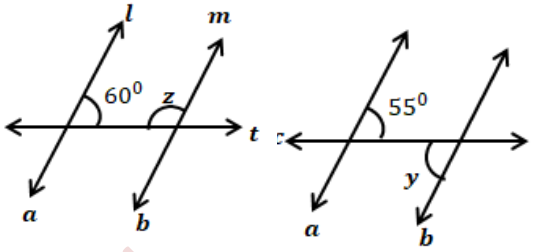
(i) Lines $l \parallel m$; t is a transversal $\angle x$=?



Sol: $\angle x = 60^0$ (Corresponding angles are equal)

(ii) Lines $a \parallel b$; c is a transversal $\angle$ y = ?

Sol: $\angle y = 55^0$ (Alternate interior angles)



(iii) l_1 , l_2 be two lines t is a transversal Is $\angle 1 = \angle 2$?

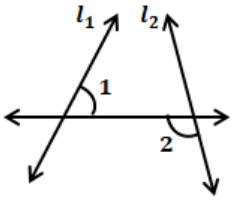
Sol: $l_1 \nparallel l_2$, so $\angle 1 \neq \angle 2$

(iv) Lines $l \parallel m$; t is a transversal $\angle$ z = ?

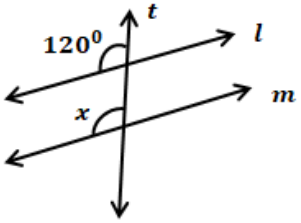
Sol: Interior angles on the same side of the transversal are supplementary

$z + 60^0 = 180^0$

$z = 180^0 - 60^0 = 120^0$

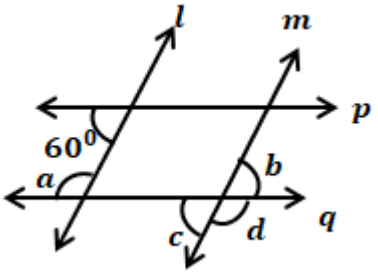


(v)) Lines $l \parallel m$; t is a transversal $\angle$ x =?



Sol: $\angle x = 120^0$ (Corresponding angles)

(vi) Lines $l \parallel m, p \parallel q$; Find a, b, c, d



Sol: $a + 60^0 = 180^0$ ($p \parallel q$, Interior angles on the same side are supplementary)

$a = 180^0 - 60^0 = 120^0$

$b = 60^0$ ($l \parallel m$; alternate interior angles)

$c = b = 60^0$ (Vertically opposite angles)

$d + b = 180^0$ (Linear pair)

$d + 60^0 = 180^0$

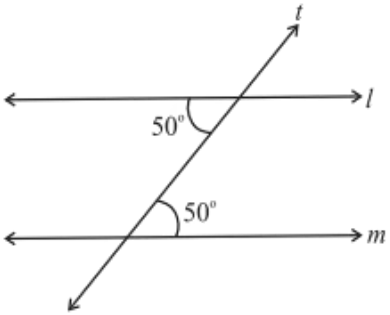
$d = 180^0 - 60^0 = 120^0$

CHECKING FOR PARALLEL LINES

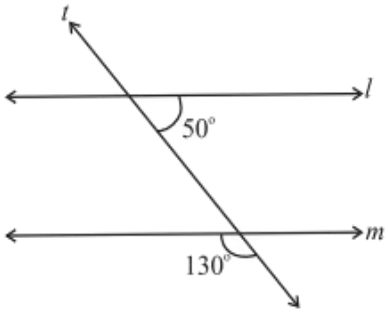
(i) When a transversal cuts two lines, such that pairs of corresponding angles are equal, then the lines have to be parallel.

(ii) When a transversal cuts two lines, such that pairs of alternate interior angles are equal, the lines have to be parallel.

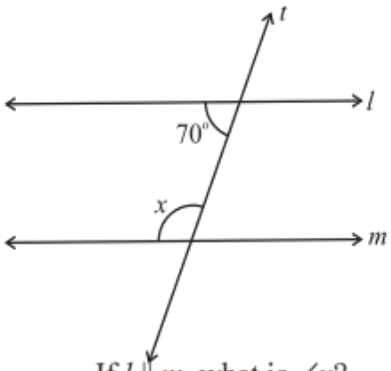
(iii) When a transversal cuts two lines, such that pairs of interior angles on the same side of the transversal are supplementary, the lines have to be parallel.



Is $l \parallel m$? Why?



Is $l \parallel m$? Why?



If $l \parallel m$, what is $\angle x$?

(i) Alternate interior angles are equal. So, $l \parallel m$.

(ii) Corresponding angles are equal. So, , $l \parallel m$

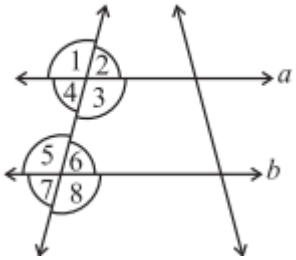
(iii) If , $l \parallel m$ then

$x + 70^0 = 180^0$ ($Interior\ angles\ on\ the\ same\ side\ of\ the\ transversal\ are\ supplementary$

$x = 180^0 - 70^0 = 110^0$

EXERCISE 5.2

1. State the property that is used in each of the following statements?



(i) $If\ a \parallel b, then \angle 1 = \angle 5$.

Sol: Corresponding angle property

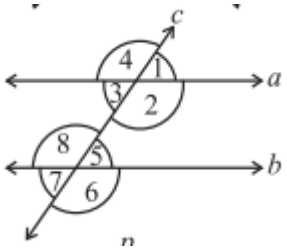
(ii) $If \angle 4 = \angle 6, then\ a \parallel b$.

Sol: Alternate interior angle property.

(iii) $If \angle 4 + \angle 5 = 180°, then\ a \parallel b$.

Sol: Interior angles on the same side of the transversal are supplementary.

2. In the adjoining figure, identify



(i) the pairs of corresponding angles.

Sol: $\angle 1, \angle 5$; $\angle 2, \angle 6$; $\angle 3, \angle 7$; $\angle 4, \angle 8$.

(ii) the pairs of alternate interior angles.

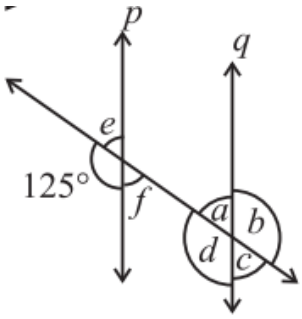
Sol: $\angle 2, \angle 8$; $\angle 3, \angle 5$.

(iii) the pairs of interior angles on the same side of the transversal.

Sol: $\angle 2, \angle 5$; $\angle 3, \angle 8$.

(iv) the vertically opposite angles.

Sol: $\angle 1, \angle 3$; $\angle 2, \angle 4$; $\angle 5, \angle 7$; $\angle 6, \angle 8$.



3. In the adjoining figure, p || q. Find the unknown angles.

Sol: $d = 125^0$ (Corresponding angles)

$b = d = 125^0$(Vertically opposite angles)

$e + 125^0 = 180^0$ (Linear pair)

$e = 180^0 - 125^0 = 55^0$

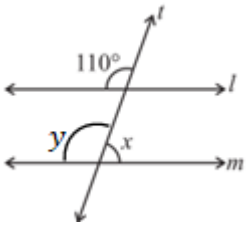
$f = e = 55^0$ (Vertically opposite angles)

$c = f = 55^0$ (Corresponding angles)

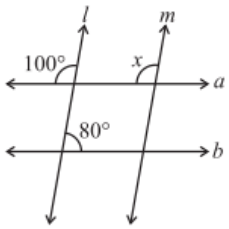
$a = c = 55^0$ (Vertically opposite angles)

$\therefore a = 55^0;\ b = 125^0\ ;\ c = 55^0;\ d = 125^0\ ;\ e = 55^0;\ f = 55^0.$

4. **Find the value of x in each of the following figures if $l \parallel m$.**



(i) $y = 110^0$(Corresponding angles)



$x + y = 180^0$(Linear pair)

$x + 110^0 = 180^0$

$x = 180^0 - 110^0$

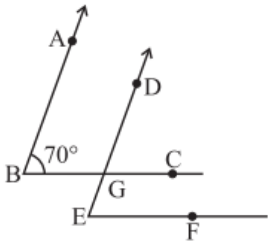
$x = 70^0$

(ii) $x = 100^0$(Corresponding angles)

5. **In the given figure, the arms of two angles are parallel. If ∠ABC = 70º, then find (i) ∠DGC (ii) ∠DEF**

Sol: (i) ∠DGC=∠ABC (Corresponding angles property)

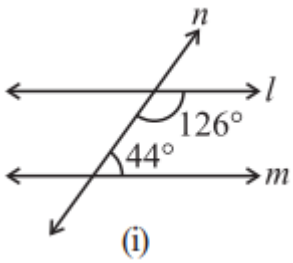
∠DGC=70^0



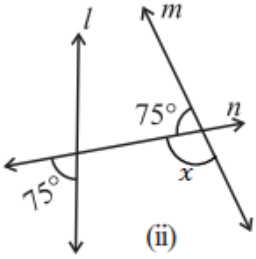
(ii) ∠DEF =∠DGC (Corresponding angles property)

∠DEF =70^0

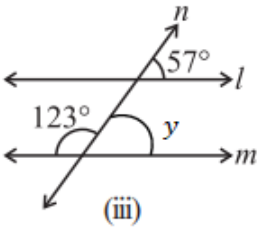
6. **In the given figures below, decide whether l is parallel to m**



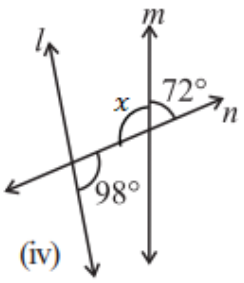
(i)



(ii)



(iii)



(iv)

(i) $44^0 + 126^0 = 170^0$

Interior angles on the same side of the transversal are not supplementary. So, $l \nparallel m$

(ii) $x + 75^0 = 180^0$(Linear pair)

$x = 180^0 - 75^0 = 105^0$

Corresponding angles 75^0 *and* 105^0 are not equal. So, $l \nparallel m$.

(iii) $y + 123^0 = 180^0$(Linear pair)

$y = 180^0 - 123^0 = 57^0$

Corresponding angles are equal (57^0). So, $l \parallel m$.

(iv) $x + 72^0 = 180^0$(Linear pair)

$x = 180^0 - 72^0 = 108^0$

Alternate interior angles 98^0 *and* 108^0 are not equal. So, $l \nparallel m$

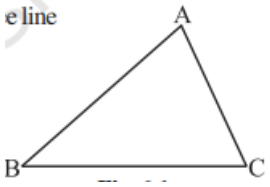
CHAPTER 6

VII-MATHEMATICS-NCERT-(2023-24)

6. The Triangle and its Properties (Notes)

REPARED BY: BALABHADRA SURESH

1. A simple closed curve made of three line segments is called a triangle.
2. Triangle has three vertices, three sides and three angles.
   Vertices: A, B, C
   Sides: AB, BC , CA
   Angles: ∠BAC, ∠ABC, ∠BCA



3. The side opposite to the vertex A is BC
   The side opposite to the vertex B is AC
   The side opposite to the vertex C is AB

TRY THESE

1. Write the six elements (i.e., the 3 sides and the 3 angles) of ΔABC
2. **Write the**

(i) Side opposite to the vertex Q of ΔPQR

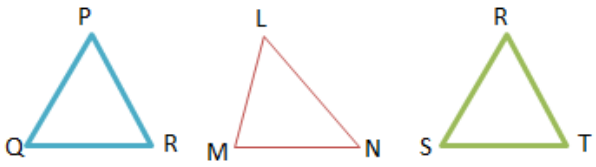
Sol: PR

(ii) Angle opposite to the side LM of ΔLMN

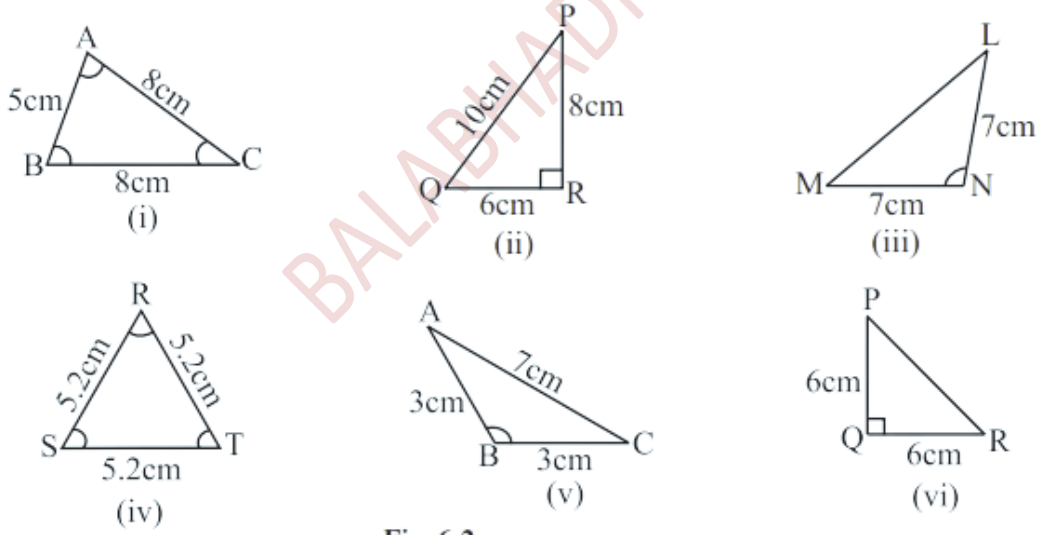
Sol: ∠N

(iii) Vertex opposite to the side RT of ΔRST

Sol: S



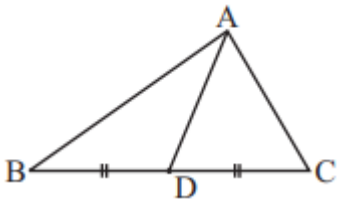
3. **Look at Fig 6.2 and classify each of the triangles according to its (a) Sides (b) Angles**



S.No	Based on sides	Based on angles
(i)	Isosceles triangle	Acute angled triangle
(ii)	Scalene triangle	Right angled triangle
(iii)	Isosceles triangle	Obtuse angled triangle
(iv)	Equilateral triangle	Acute angled triangle
(v)	Isosceles triangle	Obtuse angled triangle
(vi)	Isosceles triangle	Right angled triangle

MEDIANS OF A TRIANGLE

The line segment joining a vertex of a triangle to the mid point of its opposite side is called a median of the triangle. A triangle has 3 medians



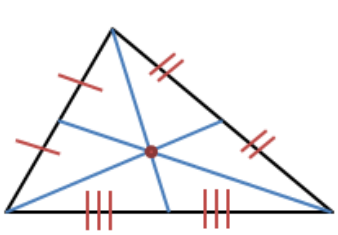
The line segment AD, joining the mid-point of BC to its opposite vertex A is called a median of the triangle.

THINK, DISCUSS AND WRITE

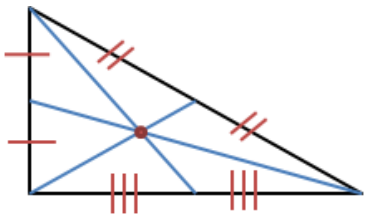
1. **How many medians can a triangle have?**

Sol:3

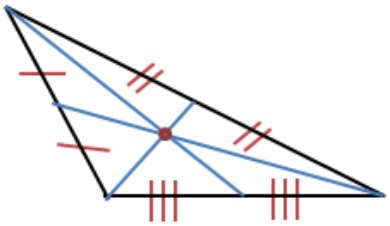
2. **Does a median lie wholly in the interior of the triangle? (If you think that this is not true, draw a figure to show such a case).**



Acute angled triangle



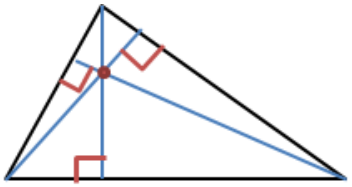
Right angled triangle



Obtuse angled triangle

ALTITUDES OF A TRIANGLE

The perpendicular line segment from a vertex of a triangle to its opposite side is called an altitude of the triangle. A triangle has 3 altitudes.



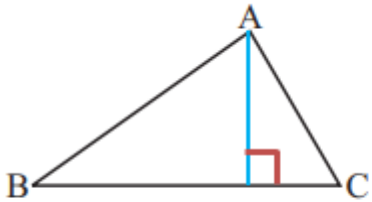
Acute angled triangle

THINK, DISCUSS AND WRITE

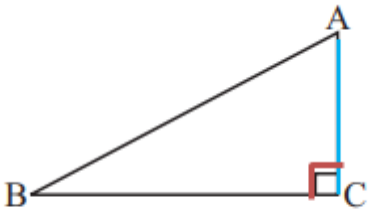
1. **How many altitudes can a triangle have?**

Sol: 3

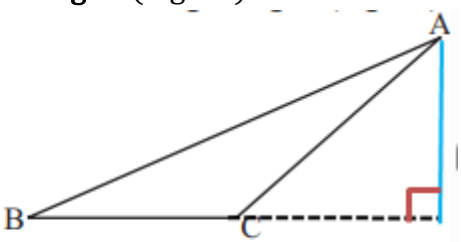
2. **Draw rough sketches of altitudes from A to BC for the following triangles** (Fig 6.6):



Acute-angled
(i)

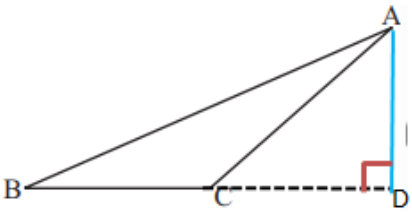


Right-angled
(ii)



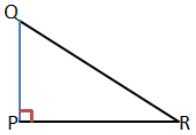
Obtuse-angled
(iii)

3. Will an altitude always lie in the interior of a triangle? If you think that this need not be true, draw a rough sketch to show such a case.



Sol: No, an altitude may lie outside of triangle also.

4. Can you think of a triangle in which two altitudes of the triangle are two of its sides?

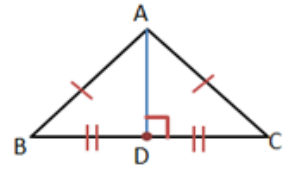


Sol: Yes, in right angled triangle two altitudes of the triangle are two of its sides.

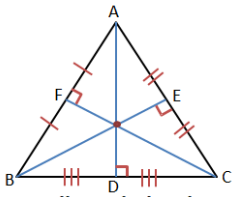
5. Can the altitude and median be same for a triangle?

Sol: Yes, in an equilateral triangle both the median and the altitude are the same.

In an isosceles triangle one altitude and median be same



Isosceles triangle



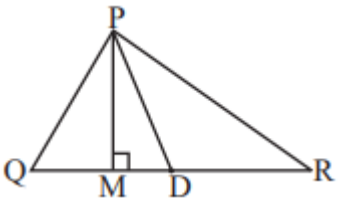
Equilateral triangle

DO THIS

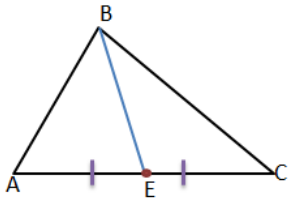
Take several cut-outs of (i) an equilateral triangle (ii) an isosceles triangle and (iii) a scalene triangle. Find their altitudes and medians. Do you find anything special about them? Discuss it with your friends.

EXERCISE 6.1

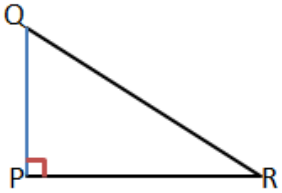
1. In Δ PQR, D is the mid-point of $\overline{QR}$
   $\overline{PM}$ is **altitude**. $\overline{PD}$ is **median**.
   Is QM = MR? NO



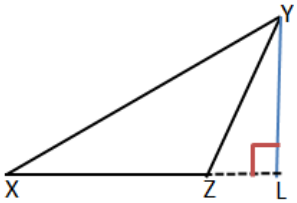
2. Draw rough sketches for the following:
   (a) In ΔABC, BE is a median.



(b) In ΔPQR, PQ and PR are altitudes of the triangle.

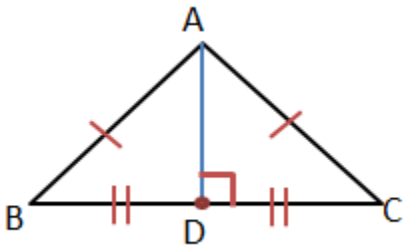


(c) In ΔXYZ, YL is an altitude in the exterior of the triangle.



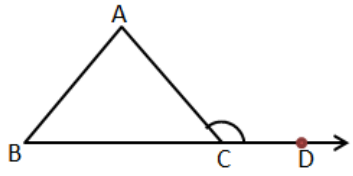
3. **Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.**

Sol:



EXTERIOR ANGLE OF A TRIANGLE AND ITS PROPERTY

An exterior angle of a triangle is formed, when a side of a triangle is produced. At each vertex, you have two ways of forming an exterior angle.



Exterior Angle Property of a triangle:

The measure of any exterior angle of a triangle is equal to the sum of the measures of its interior opposite angles

Given: Consider ΔABC. ∠ACD is an exterior angle.

To Show: m ∠ACD = m ∠A + m ∠B

Proof: Through C draw $\overline{CE}$, parallel to $\overline{BA}$.

∠1 = ∠x ($\overline{BA}$ ∥ $\overline{CE}$and $\overline{AC}$ is a transversal. Therefore, alternate angles should be equal)

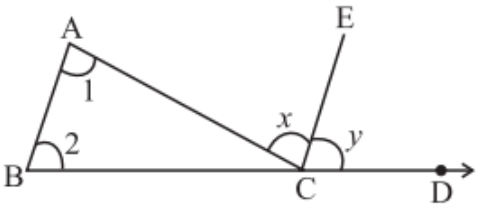
∠2 = ∠y ($\overline{BA}$ ∥ $\overline{CE}$and $\overline{BD}$ is a transversal. Therefore, corresponding angles should be equal)

$\angle 1 + \angle 2 = \angle x + \angle y$

∠x + ∠y = m ∠ACD (From Fig)

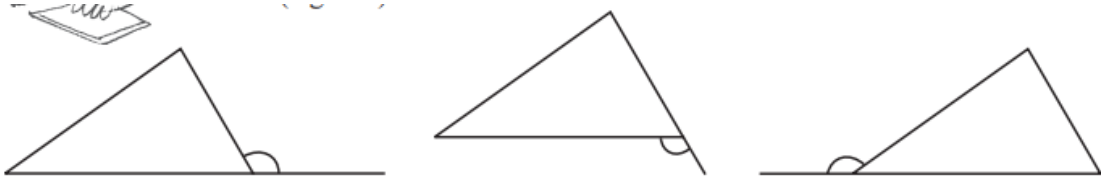
Hence, ∠ACD=∠1 + ∠2

m ∠ACD = m ∠A + m ∠B

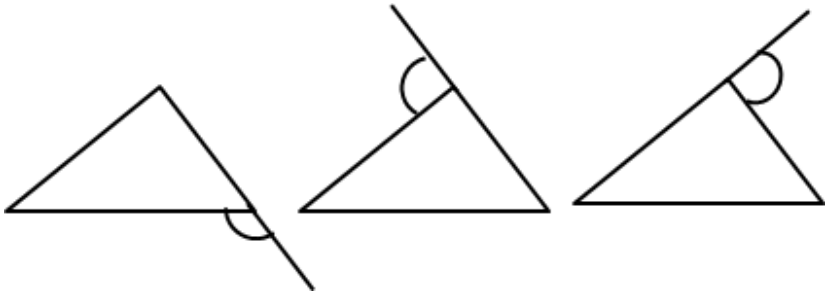


THINK, DISCUSS AND WRITE

1. **Exterior angles can be formed for a triangle in many ways. Three of them are shown here There are three more ways of getting exterior angles. Try to produce those rough sketches**



Sol:



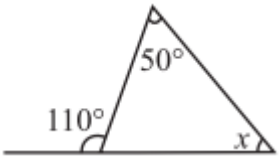
2. **Are the exterior angles formed at each vertex of a triangle equal?**

Sol: No

3. **What can you say about the sum of an exterior angle of a triangle and its adjacent interior angle?**

Sol: The sum of an exterior angle of a triangle and its adjacent interior angle=180^0

Example 1:Find angle x in Fig 6.11.



Solution: Sum of interior opposite angles = Exterior angle

$$50° + x = 110°$$

$$x = 110° - 50°$$

$$x = 60°$$

THINK, DISCUSS AND WRITE

1. What can you say about each of the interior opposite angles, when the exterior angle is (i) a right angle? (ii) an obtuse angle? (iii) an acute angle?
2. **Can the exterior angle of a triangle be a straight angle?**

Sol: No

TRY THESE

1. **An exterior angle of a triangle is of measure 70º and one of its interior opposite angles is of measure 25º. Find the measure of the other interior opposite angle**

Sol: Sum of interior opposite angles = Exterior angle

$$25° + x = 70°$$

$$x = 70° - 25°$$

$$x = 45°$$

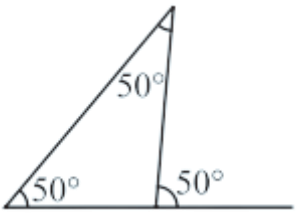
The other interior opposite angle=45^0

2. **The two interior opposite angles of an exterior angle of a triangle are 60º and 80º. Find the measure of the exterior angle.**

Sol: Exterior angle= Sum of interior opposite angles

$=60^0+80^0=140^0$

3. Is something wrong in this diagram (Fig 6.12)? Comment



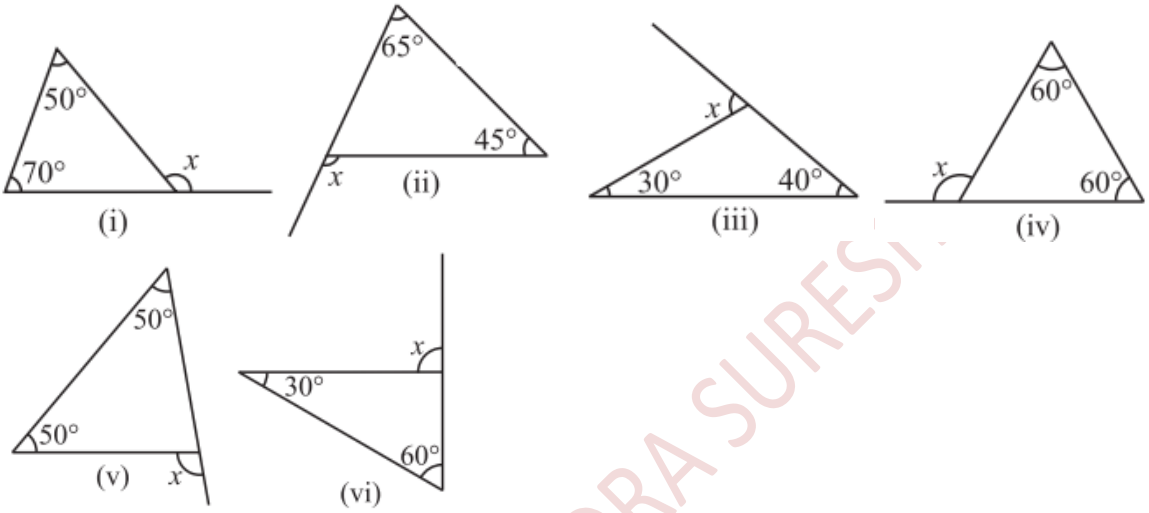
Sol: Exterior angle$=50^0$

Sum of interior opposite angles$=50^0+50^0=100^0$

Exterior angle$\neq$ Sum of interior opposite angles

EXERCISE 6.2

1. Find the value of the unknown exterior angle x in the following diagrams:



Sol: Exterior angle= Sum of interior opposite angles

$(i)\ x = 50^0 + 70^0 = 120^0$

$(ii)\ x = 65^0 + 45^0 = 110^0$

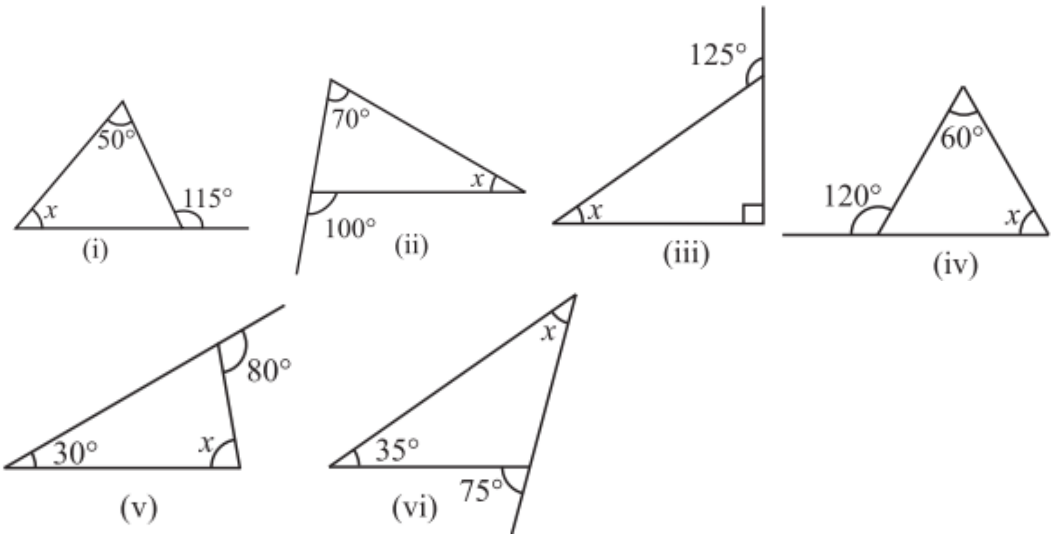
$(iii)\ x = 30^0 + 40^0 = 70^0$

$(iv)\ x = 60^0 + 60^0 = 120^0$

$(v)\ x = 50^0 + 50^0 = 100^0$

$(vi)\ x = 30^0 + 60^0 = 90^0$

2. Find the value of the unknown interior angle x in the following figures:



Sol: Sum of interior opposite angles = Exterior angle

(i) $x + 50^0 = 115^0$

$x = 115^0 - 50^0$

$x = 65^0$

(ii) $x + 70^0 = 100^0$

$x = 100^0 - 70^0$

$x = 30^0$

(iii) $x + 90^0 = 125^0$

$x = 125^0 - 90^0$

$x = 35^0$

(iv) $x + 60^0 = 120^0$

$x = 120^0 - 60^0$

$x = 60^0$

(v) $x + 30^0 = 80^0$

$x = 115^0 - 50^0$

$x = 65^0$

(vi) $x + 35^0 = 75^0$

$x = 75^0 - 35^0$

$x = 40^0$

ANGLE SUM PROPERTY OF A TRIANGLE

The total measure of the three angles of a triangle is 180°

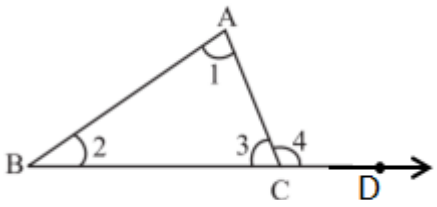
Given :∠1, ∠2, ∠3 are angles of ΔABC and ∠4 is the exterior angle when BC is extended to D.

Proof: ∠1 + ∠2 = ∠4 (by exterior angle property)

∠1 + ∠2 + ∠3 = ∠4 + ∠3 (adding ∠3 to both the sides)

But ∠4 + ∠3=180° (linear pair)

∴ ∠1 + ∠2 + ∠3 = 180°



Example 2: In the given figure (Fig 6.18) find m∠P

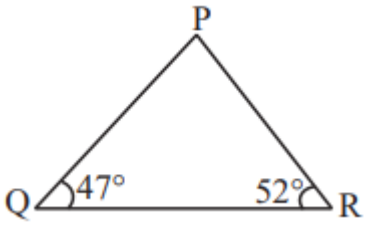
Solu: By angle sum property of a triangle,

∠P + 47° + 52° = 180°

∠P + 99° = 180°

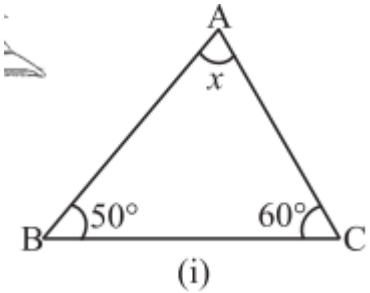
∠P = 180° – 99°

m∠P = 81°

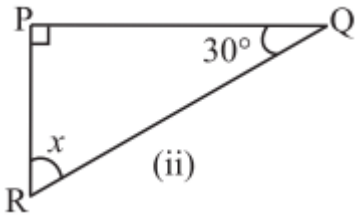


EXERCISE 6.3

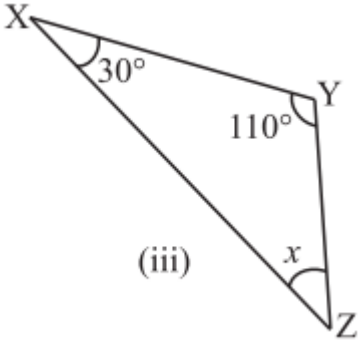
1. Find the value of the unknown x in the following diagrams:



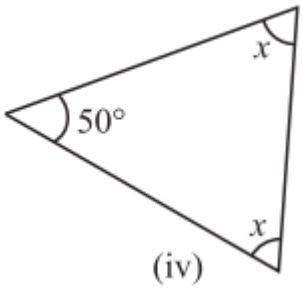
(i)



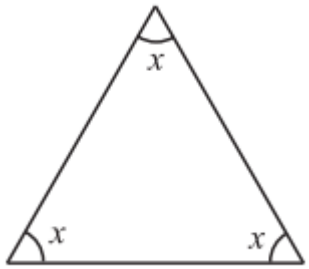
(ii)



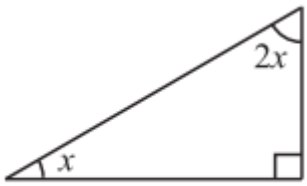
(iii)



(iv)



(v)



(vi)

Sol: Sum of three angles in a triangle=180^0

(i) $x + 50^0 + 60^0 = 180^0$

$x + 110^0 = 180^0$

$x = 180^0 - 110^0$

$x = 70^0$

(ii) $x + 30^0 + 90^0 = 180^0$

$x + 120^0 = 180^0$

$x = 180^0 - 120^0$

$x = 60^0$

(iii) $x + 110^0 + 30^0 = 180^0$

$x + 140^0 = 180^0$

$x = 180^0 - 140^0$

$x = 40^0$

(iv) $x + x + 50^0 = 180^0$

$2x = 180^0 - 50^0 = 130^0$

$x = \frac{130^0}{2}$

$x = 65^0$

(v) $x + x + x = 180^0$

$3x = 180^0$

$x = \frac{180^0}{3} = 60^0$

$x = 70^0$

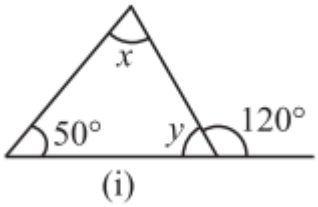
(vi) $x + 2x + 90^0 = 180^0$

$3x = 180^0 - 90^0$

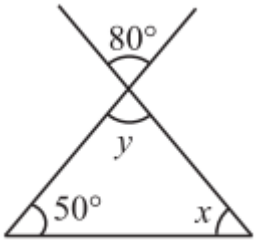
$x = \frac{90^0}{3}$

$x = 30^0$

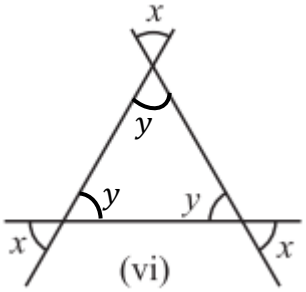
2. Find the values of the unknowns x and y in the following diagrams:



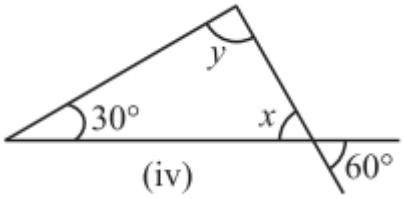
(i)



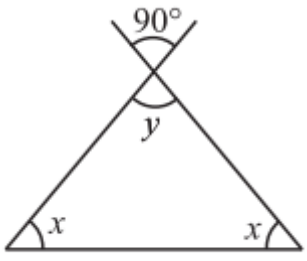
(ii)



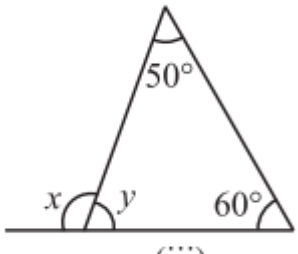
(vi)



(iv)



(v)



(iii)

Sol:

(i) $y + 120^0 = 180^0$(Linear pair)

$y = 180^0 - 120^0 = 60^0$

$x + 50^0 = 120^0$(Exterior angle property)

$x = 120^0 - 50^0 = 70^0$

(ii) $y = 80^0$(Vetically opposite angles)

$x + y + 50^0 = 180^0$(Angle sum property of triangle)

$x + 80^0 + 50^0 = 180^0$

$x + 130^0 = 180^0$

$x = 180^0 - 130^0$

$x = 50^0$

(iii) $x = 50^0 + 60^0$ (Exterior angle property)

$x = 110^0$

$x + y = 180^0$(Linear pair)

$110^0 + y = 180^0$

$y = 180^0 - 110^0$

$y = 70^0$

(iv) $x = 60^0$(Vetically opposite angles)

$x + y + 30^0 = 180^0$(Angle sum property of triangle)

$60^0 + y + 30^0 = 180^0$

$y + 90^0 = 180^0$

$y = 180^0 - 90^0 = 90^0$

(v) $y = 90^0$(Vetically opposite angles)

$x + x + y = 180^0$(Angle sum property of triangle)

$2x + 90^0 = 180^0$

$2x = 180^0 - 90^0$

$2x = 90^0$

$x = \frac{90^0}{2} = 45^0$

(vi) $x = y$(Vetically opposite angles)

$y + y + y = 180^0$(Angle sum property of triangle)

$3y = 180^0$

$y = \frac{180^0}{3} = 60^0$

$x = 60^0$ and $y = 60^0$

TRY THESE

1. **Two angles of a triangle are 30º and 80º. Find the third angle.**

Sol: Let third angle=x

$x + 30^0 + 80^0 = 180^0$(Angle sum property of triangle)

$x + 110^0 = 180^0$

$x = 180^0 - 110^0 = 70^0$

2. **One of the angles of a triangle is 80º and the other two angles are equal. Find the measure of each of the equal angles.**

Sol: Let the other two angles are x, x

$x + x + 80^0 = 180^0$(Angle sum property of triangle)

$2x + 80^0 = 180^0$

$2x = 180^0 - 80^0 = 100^0$

$2x = 100^0$

$x = \frac{100^0}{2} = 50^0$

3. **The three angles of a triangle are in the ratio 1:2:1. Find all the angles of the triangle. Classify the triangle in two different ways.**

Sol: Let the angles are $x, 2x, x$

$x + 2x + x = 180^0$(Angle sum property of triangle)

$4x = 180^0$

$x = \frac{180^0}{4} = 45^0$

The angles are $45^0, 2 \times 45^0, 45^0$ $i.e$ $45^0, 90^0, 45^0$

The given triangle is an isosceles triangle and right angled triangle.

THINK, DISCUSS AND WRITE

1. **Can you have a triangle with two right angles?**

Sol: No

2. **Can you have a triangle with two obtuse angles?**

Sol: No

3. **Can you have a triangle with two acute angles?**

Sol: Yes

4. **Can you have a triangle with all the three angles greater than 60º?**

Sol: No

5. **Can you have a triangle with all the three angles equal to 60º?**

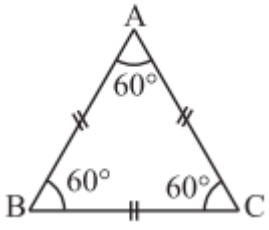
Sol: Yes, in equilateral triangle all the three angles equal to 60º

6. **Can you have a triangle wtih all the three angles less than 60º?**

Sol: No.

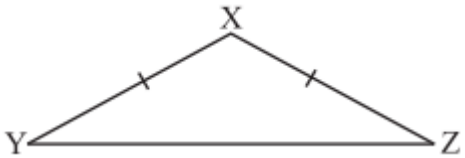
TWO SPECIAL TRIANGLES: EQUILATERAL AND ISOSCELES

A triangle in which all the three sides are of equal lengths is called an equilateral triangle.



A triangle in which two sides are of equal lengths is called an **isosceles triangle**.

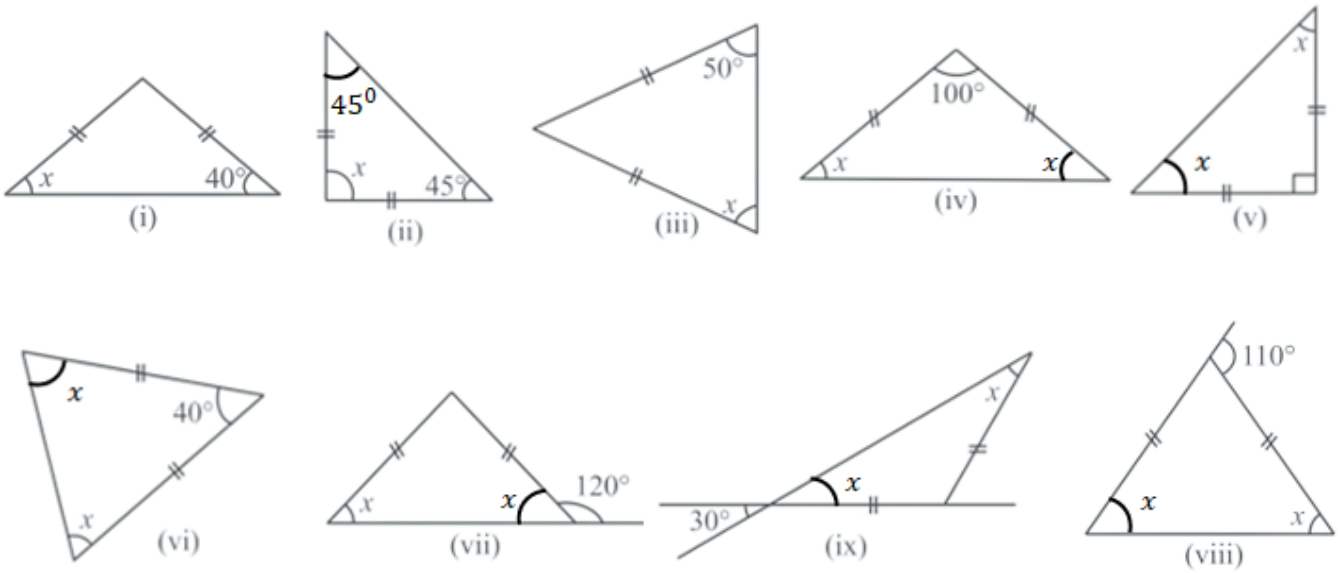
In an isosceles triangle base angles opposite to the equal sides are equal.



$XY = XZ \Rightarrow \angle Y = \angle Z$

TRY THESE

1. Find angle x in each figure:



Sol: Equal sides opposite angles are equal.

$(i)\quad x = 40^0$

$(ii)\quad x + 45^0 + 45^0 = 180^0$

$x + 90^0 = 180^0$

$x = 180^0 - 90^0 = 90^0$

$(iii)\ x = 50^0$

$(iv)\ x + x + 100^0 = 180^0$

$2x = 180^0 - 100^0 = 80^0$

$x = \frac{80^0}{2} = 40^0$

$(v)\quad x + x + 90^0 = 180^0$

$2x = 180^0 - 90^0 = 90^0$

$x = \frac{90^0}{2} = 45^0$

$(vi)\ x + x + 40^0 = 180^0$

$2x = 180^0 - 40^0 = 140^0$

$x = \frac{140^0}{2} = 70^0$

$(vii)\ x + 120^0 = 180^0$ (Linear pair)

$x = 180^0 - 120^0$

$x = 60^0$

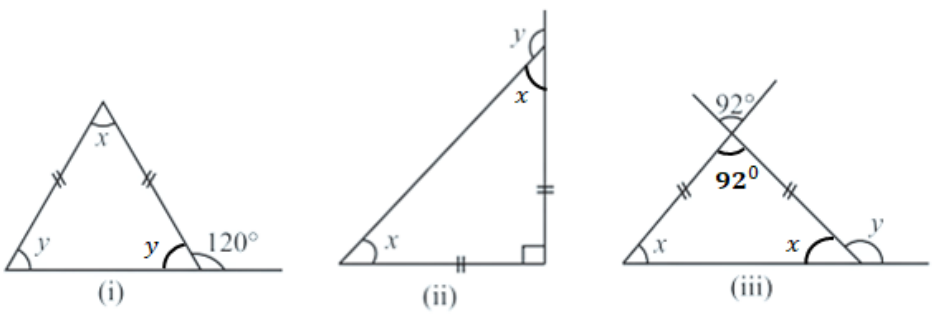
$(viii)\ x + x = 110^0$ (Exterior angle property)

$2x = 110^0$

$x = \frac{110^0}{2} = 55^0$

$(ix)\quad x = 30^0$

2. Find angles x and y in each figure.



Sol:

(i) $y + 120^0 = 180^0 (Linear\ pair)$

$y = 180^0 - 120^0 = 60^0$

$x + y = 120^0 (\ Exterior\ angle\ property)$

$x + 60^0 = 120^0$

$x = 120^0 - 60^0$

$x = 60^0$

(ii) $x + x + 90^0 = 180^0$ (Angle sum property)

$2x + 90^0 = 180^0$

$2x = 180^0 - 90^0$

$2x = 90^0$

$x = \frac{90^0}{2} = 45^0$

$y = x + 90^0$(Exterior angle property)

$y = 45^0 + 90^0 = 135^0$

(iii) $x + x + 92^0 = 180^0$ (Angle sum property)

$2x + 92^0 = 180^0$

$2x = 180^0 - 92^0$

$2x = 88^0$

$x = \frac{88^0}{2} = 44^0$

$x + y = 180^0$(Linear pair)

$44^0 + y = 180^0$

$y = 180^0 - 44^0 = 136^0$

SUM OF THE LENGTHS OF TWO SIDES OF A TRIANGLE

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

The sum of the lengths of any two sides of a triangle is greater than the third side

Example 3: Is there a triangle whose sides have lengths 10.2 cm, 5.8 cm and 4.5 cm?

Sol: 4.5 + 5.8 =10.3>10.2

5.8 + 10.2 =16> 4.5

10.2 + 4.5=14.7 > 5.8

The sum of the lengths of any two sides would be greater than the length of the third side.

Therefore, the triangle is possible.

Example 4: The lengths of two sides of a triangle are 6 cm and 8 cm. Between which two numbers can length of the third side fall?

Sol: The third side has to be less than the sum of the two sides.

The third side is thus, less than 8 + 6 = 14 cm

The third side has to be greater than the difference of the two sides. The third side is thus, greater than 8 - 6 = 2 cm

The length of the third side could be any length greater than 2 and less than 14 cm

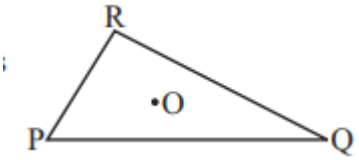
EXERCISE 6.4

1. **Is it possible to have a triangle with the following sides?**

(i) 2 cm, 3 cm, 5 cm

Sol: 2 cm+3 cm= 5 cm

The triangle is not possible



(ii) 3 cm, 6 cm, 7 cm

Sol: 3+6=9>7

3+7=10>6

6+7=13>3

The sum of the lengths of any two sides is greater than the length of the third side.

Therefore, the triangle is possible.

(iii) 6 cm, 3 cm, 2 cm

Sol: 3+2=5<6

The triangle is not possible

2. Take any point O in the interior of a triangle PQR. Is

(i) OP + OQ > PQ?

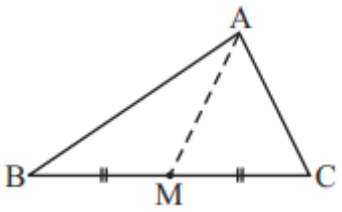
Sol: Yes.

(ii) OQ + OR > QR?

Sol: Yes.

iii) OR + OP > RP?

Sol: Yes.



3. AM is a median of a triangle ABC.

Is AB + BC + CA > 2 AM?

(Consider the sides of triangles ΔABM and ΔAMC.)

Sol: In ΔABM, AB+BM>AM →(1)

In ΔACM, MC+ CA>AM→(2)

From (1)+(2)

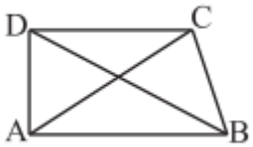
AB+BM+ MC+ CA >AM+ AM

AB + BC + CA > 2 AM (BM+MC=BC)

Hence, given statement is true.

4. ABCD is a quadrilateral.

Is AB + BC + CD + DA > AC + BD?



Sol: In ΔABC, AB+BC>AC →(1)

In ΔBCD, BC+CD>BD→(2)

In ΔDCA, CD+DA>AC →(3)

In ΔDAB, DA+AB>BD→(4)

Adding (1),(2),(3),(4)

AB+BC+ BC+CD+ CD+DA+ DA+AB> AC+ BD+ AC+ BD

2AB+2BC2+2CD+2DA>2 AC+ 2BD

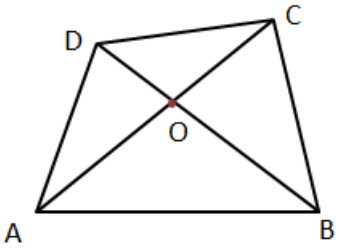
$2(AB+BC+CD+DA)>2(AC+BD)$

$AB+BC+CD+DA> AC+BD$

Hence, given statement is true.

5. **ABCD is quadrilateral. Is**
$AB + BC + CD + DA < 2 (AC + BD)$?

Sol: The sum of the lengths of any two sides is greater than the length of the third side.



In ΔAOB, $OA+OB>AB \rightarrow (1)$

In ΔBOC, $OB+OC>BC \rightarrow (2)$

In ΔCOD, $OC+OD>CD \rightarrow (3)$

In ΔDOA, $OD+OA>DA \rightarrow (4)$

From $(1)+(2)+(3)+(4)$

$OA+OB+ OB+OC+ OC+OD+ OD+OA>AB+BC+CD+DA$

$2OA+2OB+2OC+2OD>AB+BC+CD+DA$

$2(OA+OB+ OC+OD) >AB+BC+CD+DA$

$2(AC+BD) >AB+BC+CD+DA$

$AB+BC+CD+DA<2(AC+BD)$

6. **The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?**

Sol: The third side has to be less than the sum of the two sides.

The third side is thus, less than $12 + 15 = 27$ cm

The third side has to be greater than the difference of the two sides.

The third side is thus, greater than $15 - 12 = 3$ cm

The length of the third side could be any length greater than 3cm and less than 27 cm.

THINK, DISCUSS AND WRITE

1. **Is the sum of any two angles of a triangle always greater than the third angle?**

Sol: No.

RIGHT-ANGLED TRIANGLES AND PYTHAGORAS PROPERTY

In a right-angled triangle, the square on the hypotenuse = sum of the squares on the legs.

If the Pythagoras property holds, the triangle must be right-angled.

Example 5:Determine whether the triangle whose lengths of sides are 3 cm, 4 cm, 5 cm is a right-angled triangle

Solu: $3^2 = 3 \times 3 = 9$; $4^2 = 4 \times 4 = 16$; $5^2 = 5 \times 5 = 25$

$3^2 + 4^2 = 9 + 16 = 25 = 5^2$

We find $3^2 + 4^2 = 5^2$

Therefore, the triangle is right-angled.

Example 6: Δ ABC is right-angled at C. If AC = 5 cm and BC = 12 cm find the length of AB

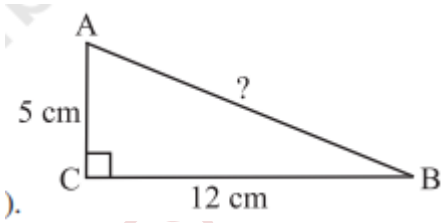
Sol: By Pythagoras property,

$AB^2 = AC^2 + BC^2$

$= 5^2 + 12^2 = 25 + 144 = 169 = 13^2$

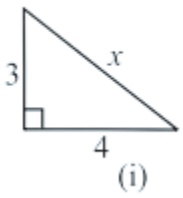
So, AB $= 13$

The length of AB is 13 cm

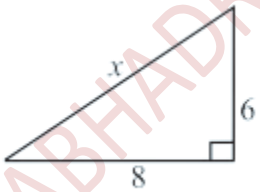


TRY THESE

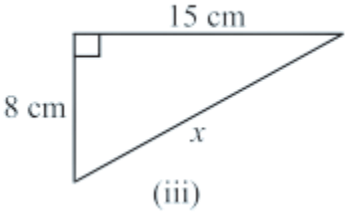
Find the unknown length x in the following figures



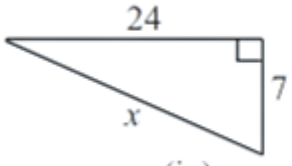
(i)



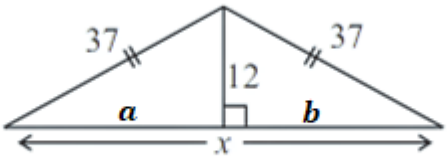
(ii)



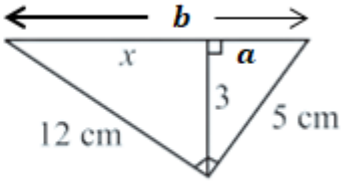
(iii)



(iv)



(v)



(vi)

Sol: By Pythagoras property,

(i) $x^2 = 3^2 + 4^2 = 9 + 16 = 25 = 5^2$

$x = 5$

(ii) $x^2 = 6^2 + 8^2 = 36 + 64 = 100 = 10^2$

$x = 10$

(iii) $x^2 = 8^2 + 15^2 = 64 + 225 = 289 = 17^2$

$x = 17$

$(iv)\ x^2 = 7^2 + 24^2 = 49 + 576 = 625 = 25^2$

$x = 25$

$(v)\ a^2 = 37^2 - 12^2 = 1369 - 144 = 1225 = 35^2$

$a = 35\ also\ b = 35$

$x = 35 + 35 = 70$

$(vi)\ b^2 = 12^2 + 5^2 = 144 + 25 = 169 = 13^2$

$b = 13$

$a^2 = 5^2 - 3^2 = 25 - 9 = 16 = 4^2$

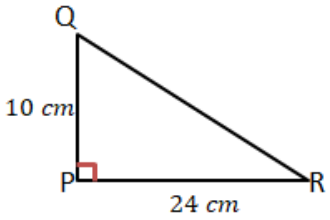
$a = 4$

$x = b - a = 13 - 4 = 9$

EXERCISE 6.5

1. PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

Sol: By Pythagoras property,



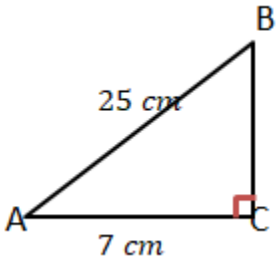
$QR^2 = PQ^2 + PR^2$

$= 10^2 + 24^2 = 100 + 576 = 676 = 26^2$

So, QR $= 26\ cm$

2. **ABC is a triangle, right-angled at C. If AB = 25 cm and AC = 7 cm, find BC.**

Sol: By Pythagoras property,



$AC^2 + BC^2 = AB^2$

$7^2 + BC^2 = 25^2$

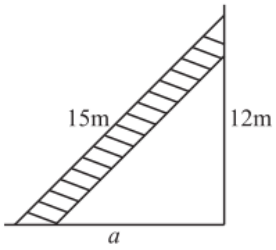
$49 + BC^2 = 625$

$BC^2 = 625 - 49 = 576 = 24^2$

BC=24 cm

3. **A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall.**

Sol: By Pythagoras property,



$a^2 + 12^2 = 15^2$

$a^2 + 144 = 225$

$a^2 = 225 - 144 = 81 = 9^2$

$a = 9\ cm$

4. **Which of the following can be the sides of a right triangle? In the case of right-angled triangles, identify the right angles**

(i) 2.5 cm,6.5 cm, 6 cm.

Sol: $(2.5)^2 + 6^2 = 6.25 + 36 = 42.25$

$(6.5)^2 = 42.25$

$(2.5)^2 + 6^2 = (6.5)^2$

Given sides form a right triangle.

(ii) 2 cm, 2 cm, 5 cm.

Sol: $2^2 + 2^2 = 4 + 4 = 8$

$5^2 = 25$

$2^2 + 2^2 \neq 5^2$

The given sides are not of a right triangle.

(iii) 1.5 cm, 2cm, 2.5 cm.

Sol: $(1.5)^2 + 2^2 = 2.25 + 4 = 6.25$

$(2.5)^2 = 6.25$

$(1.5)^2 + 2^2 = (2.5)^2$

Given sides form a right triangle

5. **A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.**

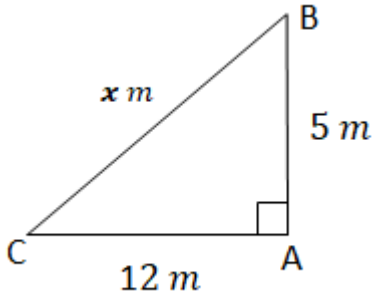
Sol: By Pythagoras property,

$BC^2 = AB^2 + AC^2$

$BC^2 = 5^2 + 12^2 = 25 + 144 = 169 = 13^2$

BC=13 m

The original height of the tree=AB+BC=5+13=18 m



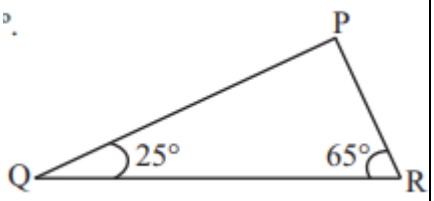
6. **Angles Q and R of a ΔPQR are 25º and 65º. Write which of the following is true:**

$(i)PQ^2 + QR^2 = RP^2 (ii)PQ^2 + RP^2 = QR^2$

$(iii)\ RP^2 + QR^2 = PQ^2$

Sol: $\angle P = 180^0 - (25^0 + 65^0) = 180^0 - 90^0 = 90^0$

$(ii)PQ^2 + RP^2 = QR^2$ *is correct*



7. **Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.**

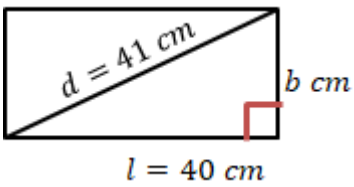
Sol: By Pythagoras property,

$l^2 + b^2 = d^2$

$40^2 + b^2 = 41^2$

$b^2 = 41^2 - 40^2 = 1681 - 1600 = 81 = 9^2$

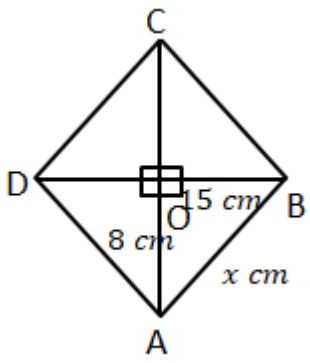
$b = 9\ cm$



The perimeter of the rectangle $= 2(l + b) = 2(40 + 9) = 2 \times 49 = 98$ cm

8. **The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.**

Sol: AC= 16 cm and BD=30 cm



Diagonal of a rhombus perpendicularly bisect each other

$$OA = OC = \frac{AC}{2} = \frac{16}{2} = 8\ cm$$

$$OB = OD = \frac{BD}{2} = \frac{30}{2} = 15\ cm$$

By Pythagoras property

$$AB^2 = OA^2 + OB^2 = 8^2 + 15^2 = 64 + 225 = 289 = 17^2$$

$$AB = 17\ cm$$

Perimeter of the rhombus=4×AB=4×17 cm=68 cm

THINK, DISCUSS AND WRITE

1. **Which is the longest side in the triangle PQR, right-angled at P?**

Sol: QR

2. **Which is the longest side in the triangle ABC, right-angled at B?**

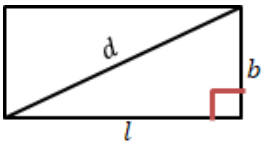
Sol: AC

3. **Which is the longest side of a right triangle?**

Sol: Opposite side of right angle (Hypotenuse)

4. **'The diagonal of a rectangle produce by itself the same area as produced by its length and breadth'– This is Baudhayan Theorem. Compare it with the Pythagoras property.**

Sol: From Baudhayan theorem



$$l^2 + b^2 = d^2$$

So, Baudhayan Theorem and Pythagoras theorem are basically same.

DO THIS

There are many proofs for Pythagoras theorem, using 'dissection' and 'rearrangement' procedure. Try to collect a few of them and draw charts explaining them.

5.

CHAPTER 7

VII-MATHEMATICS-NCERT

7. COMPARING QUANTITIES (NOTES)

REPARED BY: BALABHADRA SURESH

1. Comparing two quantities of the same kind by division is called 'Ratio'
2. The ratio of two numbers 'a' and 'b' is a÷b or $\frac{a}{b}$ and is denoted by a:b
3. **Percentage**: Percentages are numerators of fractions with denominator 100
4. Per cent is derived from Latin word 'per centum' meaning 'per hundred'
5. Per cent is represented by the symbol %

TRY THESE

1. Find the Percentage of children of different heights for the following data.

Height	Number of Children	In Fraction	In Percentage
110 cm	22	$\frac{22}{100}$	22%
120 cm	25	$\frac{25}{100}$	25%
128 cm	32	$\frac{32}{100}$	32%
130 cm	21	$\frac{21}{100}$	21%
Total	100		

2. A shop has the following number of shoe pairs of different sizes.

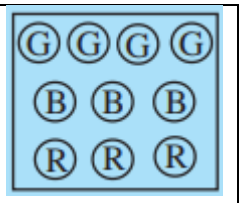
Size	Number of shoe pairs	In Fraction	In Percentage
2	20	$\frac{20}{100}$	20%
3	30	$\frac{30}{100}$	30%
4	28	$\frac{28}{100}$	28%
5	14	$\frac{14}{100}$	14%
6	8	$\frac{8}{100}$	8%
Total	100		

Percentages when total is not hundred

First we wright the fraction and next we multiply with 100 add % symbol.

TRY THESE

1. A collection of 10 chips with different colours is given. Fill the table and find the percentage of chips of each colour.

Colour	Number	Fraction	Denominator Hundred	In Percentage	
Green	4	$\frac{4}{10}$	$\frac{4\times10}{10\times10}=\frac{40}{100}$	40%	
Blue	3	$\frac{3}{10}$	$\frac{3\times10}{10\times10}=\frac{30}{100}$	30%	
Red	3	$\frac{3}{10}$	$\frac{3\times10}{10\times10}=\frac{30}{100}$	30%	
Total	10				

2. **Mala has a collection of bangles. She has 20 gold bangles and 10 silver bangles. What is the percentage of bangles of each type? Can you put it in the tabular form as done in the above example?**

Bangle	Number	Fraction	In Percentage
Gold bangles	20	$\frac{20}{30}$	$\frac{2}{3}\times100=\frac{200}{3}\%$
Silver bangles	10	$\frac{10}{30}$	$\frac{1}{3}\times100=\frac{100}{3}\%$
Total	30		

Converting Fractional Numbers to Percentage

Multiply the fraction by 100 and place the '%' symbol after it

Example1: Write $\frac{1}{3}$ as per cent.

Sol: $\frac{1}{3}=\frac{1}{3}\times100\%=\frac{100}{3}\%=33\frac{1}{3}\%$

Example 2: Out of 25 children in a class, 15 are girls. What is the percentage of girls?

Sol: Number of children=25 , girls=15

$$\text{Fraction of girls}=\frac{15}{25}$$

$$\text{Percentage of girls}=\frac{15}{25}\times100\%=15\times4\%=60\%$$

Example 3: Convert $\frac{5}{4}$ to per cent.

Sol: $\frac{5}{4}=\frac{5}{4}\times100\%=5\times25\%=125\%$

Converting Decimals to Percentage

Example 4 :Convert the given decimals to per cents:

$(a)\mathbf{0.75} = \frac{75}{100} = 75\%$

$(b)\mathbf{0.09} = \frac{9}{100} = 9\%$

$(c)\mathbf{0.2} = \frac{2}{10} = \frac{20}{100} = 20\%$

TRY THESE

1. Convert the following to per cents:

$(a)\ \frac{\mathbf{12}}{\mathbf{16}} = \frac{12}{16} \times 100\% = 3 \times 25\% = 75\%$

$(b)\mathbf{3.5} = \frac{35}{10} = \frac{35}{10} \times 100\% = 350\%$

$(c)\ \frac{\mathbf{49}}{\mathbf{50}} = \frac{49}{50} \times 100\% = 49 \times 2\% = 98\%$

$(d)\ \frac{\mathbf{2}}{\mathbf{2}} = 1 \times 100\% = 100\%$

$(e)\mathbf{0.05} = \frac{5}{100} = 5\%$

2. (i) Out of 32 students, 8 are absent. What per cent of the students are absent?

$Sol: The$ per cent of the students are absent $= \frac{8}{32} \times 100\% = 25\%$

(ii) There are 25 radios, 16 of them are out of order. What per cent of radios are out of order?

$Sol: The$ per cent of radios are out of order $= \frac{16}{25} \times 100\% = 16 \times 4\% = 64\%$

(iii) A shop has 500 items, out of which 5 are defective. What per cent are defective?

$Sol: The\ percentage\ of\ defective\ items = \frac{5}{500} \times 100\% == 1\%$

(iv) There are 120 voters, 90 of them voted yes. What per cent voted yes?

$Sol: The\ percentage\ of$ voted yes $= \frac{90}{120} \times 100\% = 3 \times 25\% = 75\%$

Converting Percentages to Fractions or Decimals

$(i)\ 1\% = \frac{1}{100} = 0.01$

$(ii)10\% = \frac{10}{100} = 0.10$

$(iii)25\% = \frac{25}{100} = 0.25$

$(iv)50\% = \frac{50}{100} = 0.50$

$(v)90\% = \frac{90}{100} = 0.90$

$(vi)125\% = \frac{125}{100} = 1.25$

$(vii)250\% = \frac{250}{100} = 2.50$

Parts always add to give a whole

All the parts that form the whole when added together gives the whole or 100%

TRY THESE

1. $35\% + 65\% = 100\%$, $64\% + 20\% + 16\% = 100\%$

$45\% = 100\% - 55\%,$ $70\% = 100\% - 30\%$

2. If 65% of students in a class have a bicycle, what per cent of the student do not have bicycles?

Sol: per cent of the student do not have bicycles=100%-65%=35%

3. We have a basket full of apples, oranges and mangoes. If 50% are apples, 30% are oranges, then what per cent are mangoes?

Example 5 :What per cent of the adjoining figure is shaded?

Sol: Half of the figure is shaded

Per cent of the figure is shaded $= \frac{1}{2} \times 100\% = 50\%$

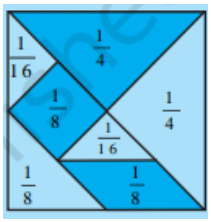
TRY THESE

What per cent of these figures are shaded?

(i) Fraction of shaded $= \frac{3}{4}$

Percentage of shaded $= \frac{3}{4} \times 100\% = 3 \times 25\% = 75\%$

(ii) Fraction of shaded $= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$



Percentage of shaded $= \frac{1}{2} \times 100\% = 50\%$

USE OF PERCENTAGES

Example 6: A survey of 40 children showed that 25% liked playing football. How many children liked playing football?

Sol: Number of children playing football=25% of 40

$= \frac{25}{100} \times 40 = \frac{1000}{100} = 10$

TRY THESE

1. Find:

$(a)\mathbf{50\%\ of\ 164} = \frac{50}{100} \times 164 = 82$

$(b)\mathbf{75\%\ of\ 12} = \frac{\cancel{75}^{3}}{\cancel{100}_{4_1}} \times \cancel{12}^{3} = 9$

$(iii)\ \mathbf{12\frac{1}{2}\ \%\ of\ 64} = \frac{25}{2}\%\ of\ 64 = \frac{25}{2 \times 100} \times 64 = 8$

2. 8% children of a class of 25 like getting wet in the rain. How many children like getting wet in the rain.

Sol: Number of children like getting wet in the rain $= 8\%\ of\ 25 = \frac{8}{100} \times 25 = 2$

Example 7: Rahul bought a sweater and saved ₹ 200 when a discount of 25% was given. What was the price of the sweater before the discount?

Sol: Let the price of sweater before the discount= x

$$25\%\ of\ x = 200$$

$$\frac{25}{100} \times x = 200$$

$$x = \frac{200 \times 100}{25} = 200 \times 4 = 800$$

1. 9 is 25% of what number?

Sol: 25% of the number = 9

$$\frac{25}{100} \times The\ number = 9$$

$$\text{The number} = \frac{9 \times 100}{25} = 9 \times 4 = 36$$

2. 75% of what number is 15?

Sol: 75% of the number = 15

$$\frac{75}{100} \times \text{The number} = 15$$

$$\text{The number} = \frac{15 \times 100}{75} = 5 \times 4 = 20$$

EXERCISE 7.1

1. Convert the given fractional numbers to per cents

$(a)\ \frac{1}{8} = \frac{1}{8} \times 100\% = \frac{25}{2}\% = 12.5\%$

$(b)\ \frac{5}{4} = \frac{5}{4} \times 100\% = 5 \times 25\% = 125\%$

$(c)\ \frac{3}{40} = \frac{3}{40} \times 100\% = \frac{15}{2}\% = 7.5\%$

$(d)\ \frac{2}{7} = \frac{2}{7} \times 100\% = \frac{200}{7}\% = 28\frac{4}{7}\%$

2. Convert the given decimal fractions to per cents.

$(a)\ 0.65 = \frac{65}{100} = 65\%$

$(b)\ 2.1 = \frac{21}{10} = \frac{210}{100} = 210\%$

$(c)\ 0.02 = \frac{2}{100} = 2\%$

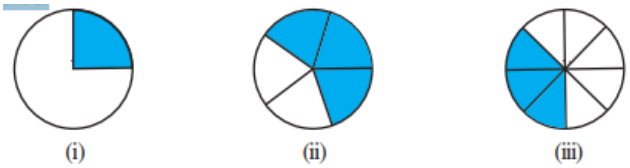
$(d)\ 12.35 = \frac{1235}{100} = 1235\%$

3. **Estimate what part of the figures is coloured and hence find the per cent which is coloured.**

$(i)\ \frac{1}{4} = \frac{1}{4} \times 100\% = 25\%$

$(ii)\ \frac{3}{5} = \frac{3}{5} \times 100\% = 3 \times 20\% = 60\%$

$(iii)\ \frac{3}{8} = \frac{3}{8} \times 100\% = \frac{75}{2}\% = 37.5\ \%$



4. Find:

$(a)\ 15\%\ of\ 250 = \frac{15}{100} \times 250 = \frac{75}{2} = 37.5$

$(b)\ 1\%\ of\ 1\ hour = \frac{1}{100} \times 60\ minutes = \frac{3}{5}\ minute. = \frac{3}{5} \times 60\ seconds = 36\ seconds$

$(\boldsymbol{c})\mathbf{20\%\ of\ ₹\ 2500} = \frac{20}{100} \times 2500 = 20 \times 25 = ₹500$

$(\boldsymbol{d})\mathbf{75\%\ of\ 1\ kg} = \frac{75}{100} \times 1\ kg = 0.75\ kg = 0.75 \times 1000\ g = 750\ g$

5. Find the whole quantity if

(a) 5% of it is 600.

Sol: 5% of x = 600

$\frac{5}{100} \times x = 600$

$x = \frac{600 \times 100}{5} = 600 \times 20 = 12000$

(b) 12% of it is ₹ 1080.

Sol: 12% *of* $x = 1080$

$\frac{12}{100} \times x = 1080$

$x = \frac{1080 \times 100}{12} = 90 \times 100 = 9000$

(c) 40% of it is 500 km.

Sol: 40% of x = 500 km

$\frac{40}{100} \times x = 500km$

$x = \frac{500 \times 100}{40} = 250 \times 5 = 1250\ km$

(d) 70% of it is 14 minutes.

Sol: 70% of $x = 14$ minutes

$\frac{70}{100} \times x = 14\ minutes$

$x = \frac{14 \times 100}{70} = 2 \times 10 = 20\ minutes$

6. Convert given per cents to decimal fractions and also to fractions in simplest forms:

$(\boldsymbol{a})\mathbf{25\%} = \frac{25}{100} = 0.25 = \frac{1}{4}$

$(\boldsymbol{b})\mathbf{150\%} = \frac{150}{100} = 1.5 = \frac{3}{4}$

$(\boldsymbol{c})\mathbf{20\%} = \frac{20}{100} = 0.2 = \frac{1}{5}$

$(\boldsymbol{d})\mathbf{5\%} = \frac{5}{100} = 0.05 = \frac{1}{20}$

7. In a city, 30% are females, 40% are males and remaining are children. What per cent are children?

Sol: Percent of children $= 100\% - (30\% + 40\%) = 100\% - 70\% = 30\%$

8. Out of 15,000 voters in a constituency, 60% voted. Find the percentage of voters who did not vote. Can you now find how many actually did not vote?

Sol: Percentage of voted voters=60%

Percentage of voters who did not vote=100%-40%=60%

The number of voters who did not vote=40% of 15,000

$= \frac{40}{1\cancel{00}} \times 150\cancel{00} = 40 \times 150 = 6{,}000$

9. Meeta saves ₹ 4000 from her salary. If this is 10% of her salary. What is her salary?

Sol: 10% of salary = ₹ 4000

$\frac{10}{100} \times$ Salary = ₹4000

Salary $= \frac{4000 \times 10\cancel{0}}{1\cancel{0}} = 4000 \times 10 = 40000$

10. A local cricket team played 20 matches in one season. It won 25% of them. How many matches did they win?

Sol: Number of matches won=25% of 20

$$= \frac{\cancel{25}^{1}}{\cancel{100}_{4}} \times \cancel{20}^{5} = 5$$

Ratios to Percents

Example 8: Reena's mother said, to make idlis, you must take two parts rice and one part urad dal. What percentage of such a mixture would be rice and what percentage would be urad dal?

Sol: Rice : Urad dal = 2 : 1.

Total parts=2+1=3

Part of rice $= \frac{2}{3}$, Part of urad dal $= \frac{1}{3}$

Percentage of rice $= \frac{2}{3} \times 100\% = \frac{200}{3}\% = 66\frac{2}{3}\%$

Percentage of rice $= \frac{1}{3} \times 100\% = \frac{100}{3}\% = 33\frac{1}{3}\%$

Example 9: If ₹ 250 is to be divided amongst Ravi, Raju and Roy, so that Ravi gets two parts, Raju three parts and Roy five parts. How much money will each get? What will it be in percentages?

Sol: Ravi: Raju:roy =2:3:5

Total parts=2+3+5=10

Amount received by Ravi $= \frac{2}{10} \times ₹250 = 2 \times 25 = ₹50$

Amount received by Raju $= \frac{3}{10} \times ₹250 = 3 \times 25 = ₹75$

Amount received by Roy $= \frac{5}{10} \times ₹250 = 5 \times 25 = ₹125$

Percentage of money received by Ravi $= \frac{2}{10} \times 100\% = 20\%$

Percentage of money received by Raju $= \frac{3}{10} \times 100\% = 30\%$

Percentage of money received by Roy $= \frac{5}{10} \times 100\% = 50\%$

TRY THESE

1. **Divide 15 sweets between Manu and Sonu so that they get 20 % and 80 % of them respectively.**

Sol: Total number of sweets=15

Sweets to be given to Manu $= 20\%$ of 15

$$= \frac{20}{100} \times 15 = 3$$

Sweets to be given to Sonu $= 80\%$ of 15

$$= \frac{80}{100} \times 15 = 4 \times 3 = 12$$

2. **If angles of a triangle are in the ratio 2 : 3 : 4. Find the value of each angle.**

Sol: Sum of angles in a triangle=180^0

The ratio of angles=2:3:4

Total parts=2+3+4=9

First angle $= \frac{2}{9} \times 180^0 = 2 \times 20^0 = 40^0$

Second angle $= \frac{3}{9} \times 180^0 = 3 \times 20^0 = 60^0$

Third angle $= \frac{4}{9} \times 180^0 = 4 \times 20^0 = 80^0$

Increase or Decrease as Per Cent

Example 10: A school team won 6 games this year against 4 games won last year. What is the per cent increase?

Sol: The increase in the number of wins (or amount of change) $= 6 - 4 = 2$.

$$\text{Percentage increase} = \frac{\text{amount of change}}{\text{original amount or base}} \times 100$$

$$= \frac{2}{4} \times 100 = 50$$

Example 11:The number of illiterate persons in a country decreased from 150 lakhs to 100 lakhs in 10 years. What is the percentage of decrease?

Sol: Original amount = the number of illiterate persons initially = 150 lakhs

Amount of change = decrease in the number of illiterate persons = 150 – 100 = 50 lakhs

$$\text{The percentage of decrease} = \frac{\text{amount of change}}{\text{original amount}} \times 100 = \frac{50}{150} \times 100 = \frac{100}{3} = 33\frac{1}{3}$$

Try These

1. **Find Percentage of increase or decrease:**

(i) Price of shirt decreased from ₹ 280 to ₹ 210

Sol: Decrease=₹ 280- ₹ 210=₹ 70

$$\text{Percentage of decrease} = \frac{\text{Decrease}}{\text{Original price}} \times 100 = \frac{\cancel{70}^{1}}{\cancel{280}_{4}} \times \cancel{100}^{25} = 25\%$$

(ii) Marks in a test increased from 20 to 30.

Sol: Increase=30-20=10

$$\text{Percentage of increase} = \frac{\text{Increase}}{\text{Initial marks}} \times 100 = \frac{\cancel{10}^{1}}{\cancel{20}_{2}} \times \cancel{100}^{50} = 50\%$$

2. **My mother says, in her childhood petrol was ₹ 1 a litre. It is ₹ 52 per litre today. By what Percentage has the price gone up?**

Sol: Increase $= 52 - 1 = ₹51$

$$\text{Percentage increase in price} = \frac{\text{Increase}}{\text{Initial price}} \times 100 = \frac{51}{1} \times 100 = 5100\%$$

PRICES RELATED TO AN ITEM OR BUYING AND SELLING

The buying price of any item is known as its cost price(CP). The price at which you sell is known as the selling price (SP).

If CP $<$ SP then you made a profit = SP – CP

If CP = SP then you are in a no profit no loss situation.

If CP $>$ SP then you have a loss = CP – SP.

$$\text{Profit percent} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$\text{Loss percent} = \frac{\text{Loss}}{\text{CP}} \times 100$$

Example 12: The cost of a flower vase is ₹ 120. If the shopkeeper sells it at a loss of 10%, find the price at which it is sold.

Sol: We are given that CP = ₹ 120 and Loss per cent = 10%

CP	SP
100	90
120	x

$100 \times x = 120 \times 90$

$$x = \frac{120 \times 90}{100} = 12 \times 9 = 108$$

SP of flower vase=₹ 108

(oR)

$$\text{Loss} = 10\% \; of \; 120 = \frac{10}{100} \times 120 = 12$$

$$\text{SP} = \text{CP} - \text{Loss} = 120 - 12 = ₹108$$

Example 13: Selling price of a toy car is ₹ 540. If the profit made by shopkeeper is 20%, what is the cost price of this toy?

Sol: SP = ₹ 540 and the Profit = 20%

CP	SP
100	120
x	540

$x \times 120 = 100 \times 540$

$$x = \frac{100 \times 540}{120} = 450$$

The cost price of this toy=₹450

TRY THESE

1. A shopkeeper bought a chair for ₹ 375 and sold it for ₹ 400. Find the gain Percentage.

Sol: C.P of chair=₹ 375, S.P of chair=₹400

Gain=S.P-C.P=₹400-₹375=₹25

$$\text{Gain Percentage} = \frac{Gain}{C.P} \times 100\% = \frac{25}{375} \times 100 = \frac{20}{3} = 6\frac{2}{3}\%$$

2. Cost of an item is ₹ 50. It was sold with a profit of 12%. Find the selling price.

Sol: C.P= ₹50 , Profit=12%

CP	SP
100	112
50	x

$x \times 100 = 50 \times 112$

$$x = \frac{50 \times 112}{100} = 56$$

Selling price=₹56

3. An article was sold for ₹ 250 with a profit of 5%. What was its cost price?

Sol: S.P=₹250 , profit = 5%.

CP	SP
100	105
x	250

$x \times 105 = 100 \times 250$

$$x = \frac{100 \times 250}{105} = \frac{5000}{21} = 238$$

4. An item was sold for ₹ 540 at a loss of 5%. What was its cost price?

Sol: S.P=₹540, loss=5%

CP	SP
100	95
x	540

$x \times 95 = 100 \times 540$

$$x = \frac{100 \times 540}{95} = \frac{20 \times 540}{19} = \frac{10800}{19} = 568.42$$

Cost price=₹568.42

CHARGE GIVEN ON BORROWED MONEY OR SIMPLE INTEREST

(i) The money you borrow is known as sum borrowed or principal(P)

(ii) The borrower has to pay some extra money is known as Interest(I)

(iii) Amount = Principal + Interest (or) A=P+I

(iv) Interest is generally given in per cent for a period of one year is known as rate of interest(R)

(v) Principal=P; Rate of interest=R; Time=T; Interest=I; Amount=A

(vi) $I = \frac{P \times T \times R}{100}$

(v) A=P+I

Example 14 :Anita takes a loan of ₹ 5,000 at 15% per year as rate of interest. Find the interest she has to pay at the end of one year.

Sol: P=₹5,000; R=15% ; T=1 year.

$$\text{Interest(I)} = \frac{P \times T \times R}{100} = \frac{5000 \times 1 \times 15}{100} = 50 \times 15 = ₹750$$

TRY THESE

1. **₹ 10,000 is invested at 5% interest rate p.a. Find the interest at the end of one year.**

Sol: P=₹10,000; R=5% ; T=1 year.

$$\text{Interest(I)} = \frac{P \times T \times R}{100} = \frac{10000 \times 1 \times 5}{100} = 1000 \times 5 = ₹5000$$

2. **₹ 3,500 is given at 7% p.a. rate of interest. Find the interest which will be received at the end of two years.**

Sol: P=₹3,500; R=7% ; T=2 year.

$$\text{Interest(I)} = \frac{P \times T \times R}{100} = \frac{3{,}500 \times 2 \times 7}{100} = 35 \times 14 = ₹490$$

3. **₹ 6,050 is borrowed at 6.5% rate of interest p.a.. Find the interest and the amount to be paid at the end of 3 years**

Sol: P=₹6,050; R=6.5% ; T=3 year.

$$\text{Interest(I)} = \frac{P \times T \times R}{100} = \frac{6{,}050 \times 3 \times 6.5}{100} = \frac{605 \times 3 \times 6.5}{10} = \frac{11797.5}{10} = ₹1179.75$$

$$\text{Amount}(A) = P + I = ₹6{,}050 + ₹1179.75 = ₹7229.75$$

4. **₹ 7,000 is borrowed at 3.5% rate of interest p.a. borrowed for 2 years. Find the amount to be paid at the end of the second year.**

Sol: P=₹7,000; R=3.5% ; T=2 year.

$$\text{Interest(I)} = \frac{\text{P} \times \text{T} \times \text{R}}{100} = \frac{3{,}500 \times 2 \times 7}{100} = 35 \times 14 = ₹490$$

$$\text{Amount}(A) = P + I = ₹7{,}000 + ₹490 = ₹7{,}490$$

Example 15: If Manohar pays an interest of ₹ 750 for 2 years on a sum of ₹ 4,500, find the rate of interest.

Sol: $\text{I} = ₹750; \text{T} = 2\ years;\ \ \text{P} = ₹\ 4{,}500$

$$\frac{\text{P} \times \text{T} \times \text{R}}{100} = \text{I}$$

$$\frac{4500 \times 2 \times R}{100} = 750$$

$$\text{R} = \frac{\cancel{750}^{\cancel{50}^{25}}}{\cancel{45}_{3} \times \cancel{2}} = \frac{25}{3} = 8\frac{1}{3}\%$$

$$\text{Rate of interest} = 8\frac{1}{3}\%$$

TRY THESE

1. **You have ₹ 2,400 in your account and the interest rate is 5%. After how many years would you earn ₹ 240 as interest.**

Sol: $\text{I} = ₹240; \text{R} = 5\%;\ \ \text{P} = ₹\ 2{,}400$

$$\frac{\text{P} \times \text{T} \times \text{R}}{100} = \text{I}$$

$$\frac{2400 \times \text{T} \times 5}{100} = 240$$

$$\text{T} = \frac{240}{24 \times 5} = 2\ years$$

2. **On a certain sum the interest paid after 3 years is ₹ 450 at 5% rate of interest per annum. Find the sum.**

Sol: $\text{I} = ₹450; \text{T} = 3\ years;\ \ \text{R} = 5\%$

$$\frac{\text{P} \times \text{T} \times \text{R}}{100} = \text{I}$$

$$\frac{P \times 3 \times 5}{100} = 450$$

$$\text{P} = \frac{450 \times 100}{15} = 30 \times 100 = ₹3000$$

EXERCISE 7.2

1. **Tell what is the profit or loss in the following transactions. Also find profit per cent or loss per cent in each case.**

(a) Gardening shears bought for ₹ 250 and sold for ₹ 325.

Sol: CP= ₹250, SP=₹325

Since CP<SP, so there is a profit.

Profit=SP-CP=325-250=₹75

$$\text{Profit percent} = \frac{Profit}{CP} \times 100\% = \frac{75}{250} \times 100 = 3 \times 10 = 30\%$$

(b) A refrigerator bought for ₹ 12,000 and sold at ₹ 13,500.

Sol: CP= ₹12,000, SP=₹13,500

Since CP<SP, so there is a profit.

Profit=SP-CP=13,500-12000=₹1500

$$\text{Profit percent} = \frac{\text{Profit}}{\text{CP}} \times 100\% = \frac{1500}{12000} \times 100 = \frac{150}{12} = \frac{25}{2} = 12\frac{1}{2}\%$$

(c) A cupboard bought for ₹ 2,500 and sold at ₹ 3,000.

Sol: CP= ₹2500, SP=₹3000

Since CP<SP, so there is a profit.

Profit=SP-CP=3000-2500=500

$$\text{Profit percent} = \frac{\text{Proft}}{\text{CP}} \times 100\% = \frac{500}{2500} \times 100 = 20\%$$

(d) A skirt bought for ₹ 250 and sold at ₹ 150

Sol: CP= ₹250, SP=₹150

Since CP>SP, so there is loss.

Profit=CP-SP=250-150=100

$$\text{Loss percent} = \frac{\text{Loss}}{\text{CP}} \times 100\% = \frac{100}{250} \times 100 = 40\%$$

2. Convert each part of the ratio to percentage:

(a) 3 : 1

Sol: Total parts=3+1=4

$$\text{Percentage of first part} = \frac{3}{4} \times 100 = 3 \times 25 = 75\%$$

$$\text{Percentage of second part} = \frac{1}{4} \times 100 = 1 \times 25 = 25\%$$

(b) 2 : 3 : 5

Sol: Total parts=2+3+5=10

$$\text{Percentage of first part} = \frac{2}{10} \times 100 = 2 \times 10 = 20\%$$

$$\text{Percentage of second part} = \frac{3}{10} \times 100 = 3 \times 10 = 30\%$$

$$\text{Percentage of third part} = \frac{5}{10} \times 100 = 5 \times 10 = 50\%$$

(c) 1:4

Sol: Total parts=1+4=5

Percentage of first part $= \frac{1}{5} \times 100 = 1 \times 20 = 20\%$

Percentage of second part $= \frac{4}{5} \times 100 = 4 \times 20 = 80\%$

(d) 1 : 2 : 5

Sol: Total parts=1+2+5=8

Percentage of first part $= \frac{1}{8} \times 100 = \frac{25}{2} = 12.5\%$

Percentage of second part $= \frac{2}{8} \times 100 = 1 \times 25 = 25\%$

Percentage of third part $= \frac{5}{8} \times 100 = \frac{125}{2} = 62.5\%$

3. The population of a city decreased from 25,000 to 24,500. Find the percentage decrease.

Sol: Decrease in papulation=25000-24500=500

The percentage decrease $= \frac{\text{Decrease in papulation}}{\text{Initial papulation}} \times 100 = \frac{500}{25000} \times 100 = 2\%$

4. Arun bought a car for ₹ 3,50,000. The next year, the price went upto ₹ 3,70,000. What was the Percentage of price increase?

Sol: Increase in price=3,70,000-3,50,000=₹20,000

Percentage of price increase $= \frac{\text{Increase in price}}{\text{Initial price}} \times 100 = \frac{20000}{350000} \times 100 = \frac{200}{35} = \frac{40}{7} = 5\frac{5}{7}\%$

5. I buy a T.V. for ₹ 10,000 and sell it at a profit of 20%. How much money do I get for it?

Sol: CP of T.V=₹10,000 , Profit percentage=20%

Profit $= 20\%$ of $10{,}000 = \frac{20}{100} \times 10000 = 20 \times 100 = ₹2000$

SP = CP+Profit=10,000+2,000=₹12,000

(or)

CP	SP
100	120
10,000	x

$x \times 100 = 10{,}000 \times 120$

$x = \frac{10{,}000 \times 120}{100} = 100 \times 120 = 12000$

SP of T.V= ₹12,000

6. Juhi sells a washing machine for ₹ 13,500. She loses 20% in the bargain. What was the price at which she bought it?

Sol: SP=₹13,500, loss=20%

CP	SP
100	80
x	13,500

$x \times 80 = 100 \times 13{,}500$

$$x = \frac{100 \times 13500}{80} = 16875$$

CP of washing machine=₹16,875

7. **(i) Chalk contains calcium, carbon and oxygen in the ratio 10:3:12. Find the percentage of carbon in chalk.**

Sol: Total parts $= 10 + 3 + 12 = 25$

The percentage of carbon in chalk $= \frac{3}{25} \times 100 = 3 \times 4 = 12\%$

(ii) If in a stick of chalk, carbon is 3g, what is the weight of the chalk stick?

Sol: 12% of weight of Stick=3g

$$\frac{12}{100} \times \text{weight of Stick} = 3 \text{ g}$$

$$\text{weight of Stick} = \frac{3 \times 100}{12} = 25g$$

8. Amina buys a book for ₹ 275 and sells it at a loss of 15%. How much does she sell it for?

Sol: CP of book=₹275, loss=15%

CP	SP
100	85
275	x

$x \times 100 = 275 \times 85$

$$x = \frac{275 \times 85}{100} = \frac{23375}{100} = 233.75$$

SP of book=₹ 233.75

9. Find the amount to be paid at the end of 3 years in each case: (a) Principal = ₹ 1,200 at 12% p.a.

Sol: P=₹1,200, T=3 y, R=12%

$$\text{Interest(I)} = \frac{\text{P} \times \text{T} \times \text{R}}{100} = \frac{1200 \times 3 \times 12}{100} = 12 \times 36 = ₹432$$

$\text{Amount} = \text{P} + \text{I} = 1200 + 432 = ₹1632$

(b) Principal = ₹ 7,500 at 5% p.a.

Sol: P=7,500, T=3y, R=5%

$$\text{Interest(I)} = \frac{\text{P} \times \text{T} \times \text{R}}{100} = \frac{7500 \times 3 \times 5}{100} = 75 \times 15 = ₹1125$$

$$\text{Amount} = \text{P} + \text{I} = 7{,}500 + 1125 = ₹8625$$

10. What rate gives ₹ 280 as interest on a sum of ₹ 56,000 in 2 years?

Sol: P=₹56,000, T=2y, I=₹ 280 ,R=?

$$\frac{\text{P} \times \text{T} \times \text{R}}{100} = I$$

$$\frac{56000 \times 2 \times R}{100} = 280$$

$$\text{R} = \frac{280}{560 \times 2} = \frac{1}{4} = 0.25\ \%$$

11. If Meena gives an interest of ₹ 45 for one year at 9% rate p.a.. What is the sum she has borrowed?

Sol: P=?, T=1y, I=₹ 45 ,R=9%

$$\frac{\text{P} \times \text{T} \times \text{R}}{100} = \text{I}$$

$$\frac{P \times 1 \times 9}{100} = 45$$

$$P = \frac{45 \times 100}{9} = 500$$

The sum Meena has borrowed=₹500

CHAPTER 8

VII-MATHEMATICS-NCERT (2023-24)

8. Rational numbers (notes)

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1. **Natural numbers**: The numbers which are used for counting are called Natural numbers and represented with letter N

   N={1,2,3,4,5,……}

2. **Whole numbers**: If '0' is added to Natural numbers then they are called Whole numbers. And is denoted by 'W'

   W={0,1,2,3,4,5,……}

3. **Integers**: Combination of positive and negative numbers including 0 are called Integers and represented by 'Z' or 'I'.

   Z={… … − 4, −3, −2, −1,0,1,2,3,4, … … … }

4. **RATIONAL NUMBER**: A number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0 is called a rational number.

   Ex: $\frac{1}{2}, \frac{-5}{7}, 1\frac{4}{5}, 0.5.\ 0.33, \ldots..$

5. Integers also rational numbers ($5 = \frac{5}{1}, -7 = \frac{-7}{1}, \ 0 = \frac{0}{2}$)
6. Rational numbers include integers and fractions.

1. Is the number $\frac{2}{-3}$ rational? Think about it.

Sol: $\frac{2}{-3}$ is rational because it is inthe form $\frac{p}{q}$, $whwre\ p = 2\ and\ q = -3(\neq 0) are\ integers.$

2. List ten rational numbers.

Sol: $\frac{1}{5}, \frac{-3}{4}, \frac{7}{9}, \frac{11}{5}, \frac{5}{-9}, \frac{25}{21}, \frac{37}{145}, 0, -5, 7.$

EQUIVALENT RATIONAL NUMBERS:

By multiplying or dividing the numerator and denominator of a rational number by the same non-zero integer we get equivalent rational number to the given.

Ex: $\frac{10}{-15} = \frac{10 \div (-5)}{-15 \div (-5)} = \frac{-2}{3}$; $\frac{-3}{7} = \frac{-3 \times 2}{7 \times 2} = \frac{-6}{14}$

TRY THESE

Fill in the boxes

(i) $\frac{5}{4}=\frac{5\times 4}{4\times 4}=\frac{5\times 5}{4\times 5}=\frac{5\times(-3)}{4\times(-3)}$

$\frac{5}{4}=\frac{20}{16}=\frac{25}{20}=\frac{-15}{-12}$

(*ii*) $\frac{-3}{7}=\frac{-3\times 2}{7\times 2}=\frac{-3\times(-3)}{7\times(-3)}=\frac{-3\times 2}{7\times 2}$

$\frac{-3}{7}=\frac{-6}{14}=\frac{9}{-21}=\frac{-6}{14}$

POSITIVE AND NEGATIVE RATIONAL NUMBERS

(i) Both the numerator and denominator of a rational number are positive integers (or negative) is called a positive rational number.

$\frac{2}{3},\frac{5}{7},\frac{11}{45},\frac{-5}{-7},\frac{-4}{-9}\cdots$

(ii) Either numerator or denominator is negative integer is called negative rational number.

$\frac{-2}{3},\frac{5}{-7},\frac{-11}{45},\cdots$

(iii) The number 0 is neither a positive nor a negative rational number.

TRY THESE

Which of these are negative rational numbers?

(i) $\frac{-2}{3}$ (Negative rational number)

(ii) $\frac{5}{7}$ (Positive rational number)

(iii) $\frac{3}{-5}$ (Negative rational number)

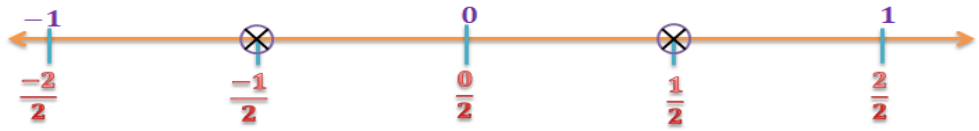
(iv) 0 (Neither positive nor negative)

(v) $\frac{6}{11}$ (Positive rational number)

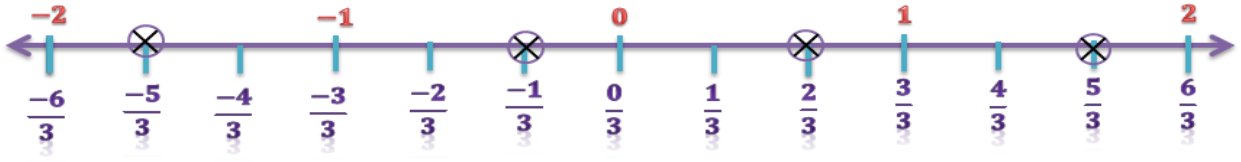
(vi) $\frac{-2}{-9}=\frac{2}{9}$ (Positive rational number)

RATIONAL NUMBERS ON A NUMBER LINE

1. Represent $\frac{1}{2}$ and $\frac{-1}{2}$ on number line.



2. Represent $\frac{-1}{3},\frac{-5}{3},\frac{2}{3}$ and $\frac{5}{3}$ on number line.



RATIONAL NUMBERS IN STANDARD FORM

A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and denominator have no common factor other than 1.

Thus, to reduce the rational number to its standard form, we divide its numerator and denominator by their HCF.

If there is negative sign in the denominator, divide by '– HCF'.

Example1: Reduce $\frac{-45}{30}$ to the standard form.

Sol: $\frac{-45}{30} = \frac{-45 \div 3}{30 \div 3} = \frac{-15 \div 5}{10 \div 5} = \frac{-3}{2}$

Example 2: Reduce to standard form:

(i) $\frac{36}{-24}$

Sol: HCF of $36,24 = 12$

$$\frac{36}{-24} = \frac{36 \div (-12)}{-24 \div (-12)} = \frac{-3}{2}$$

(ii) $\frac{-3}{-15}$

Sol: HCF of $3,15 = 3$

$$\frac{-3}{-15} = \frac{-3 \div (-3)}{-15 \div (-3)} = \frac{1}{5}$$

TRY THESE

Find the standard form of

(i) $\frac{-18}{45}$

Sol: HCF of $18,45 = 9$

$$\frac{-18}{45} = \frac{-18 \div 9}{45 \div 9} = \frac{-2}{5}$$

(ii) $\frac{-12}{18}$

Sol: HCF of $12,18 = 6$

$$\frac{-12}{18} = \frac{-12 \div 6}{18 \div 6} = \frac{-2}{3}$$

COMPARISON OF RATIONAL NUMBERS

(i) To compare rational numbers reduce them to their standard forms and then compare them.

(ii) To compare two negative rational numbers, we compare them ignoring their negative signs and then reverse the order.

(iii) A negative rational number will always be less than a positive rational number.

Exmple 3 : Do $\frac{4}{-9}$ and $\frac{-16}{36}$ represent the same rational number?

Sol: $\frac{4}{-9}=\frac{4\times(-4)}{-9\times(-4)}=\frac{-16}{36}$

$So,\frac{4}{-9}$ and $\frac{-16}{36}$ represent the same rational number.

RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS.

We can find unlimited number of rational numbers between any two rational numbers.

1. Find five rational numbers between $\frac{-5}{7}$ and $\frac{-3}{8}$.

Sol: $\frac{-5}{7}=\frac{-5\times8}{7\times8}=\frac{-40}{56}\ and\ \frac{-3}{8}=\frac{-3\times7}{8\times7}=\frac{-21}{56}$ $[LCM\ of\ 7,8=56]$

$\frac{-40}{56}<\frac{-21}{56}$

$\frac{-40}{56}<\frac{-39}{56}<\frac{-38}{56}<\frac{-37}{56}<\frac{-36}{56}<\frac{-35}{56}<\frac{-34}{56}<\cdots\cdots<\frac{-21}{56}$

Five rational numbers between $\frac{-5}{7}$ and $\frac{-3}{8}$ $are\ \frac{-39}{56},\frac{-38}{56},\frac{-37}{56},\frac{-36}{56},\frac{-35}{56}$

Example 4: List three rational numbers between – 2 and – 1.

Sol: $-2<-1$

$\frac{-2}{1}\times\frac{5}{5}<\frac{-1}{1}\times\frac{5}{5}$

$\frac{-10}{5}<\frac{-5}{5}$

$\frac{-10}{5}<\frac{-9}{5}<\frac{-8}{5}<\frac{-7}{5}<\frac{-6}{5}<\frac{-5}{5}$

The three rational numbers between – 2 and – 1 are $\frac{-9}{5},\frac{-8}{5},\frac{-7}{5}$.

Example 5:Write four more numbers in the following pattern:

$\frac{-1}{3},\frac{-2}{6},\frac{-3}{9},\frac{-4}{12},\cdots$

Sol: $\frac{-1}{3},\frac{-1\times2}{3\times2}=\frac{-2}{6},\frac{-1\times3}{3\times3}=\frac{-3}{9},\frac{-1\times4}{3\times4}=\frac{-4}{12}$

The other numbers are

$$\frac{-1\times5}{3\times5}=\frac{-5}{15},\quad \frac{-1\times6}{3\times6}=\frac{-6}{18},\quad \frac{-1\times7}{3\times7}=\frac{-7}{21},\quad \frac{-1\times8}{3\times8}=\frac{-8}{24}$$

EXERCISE 8.1

1. List five rational numbers between:

(i) -1 and 0

Sol: $-1<0$

$$\frac{-1}{1}\times\frac{6}{6}<\frac{0}{1}\times\frac{6}{6}$$

$$\frac{-6}{6}<\frac{0}{6}$$

$$\frac{-6}{6}<\frac{-5}{6}<\frac{-4}{6}<\frac{-3}{6}<\frac{-2}{6}<\frac{-1}{6}<\frac{0}{6}$$

The five rational numbers between – 1 and 0 are $\frac{-5}{6},\frac{-4}{6},\frac{-3}{6},\frac{-2}{6},\frac{-1}{6}$.

(ii) -2 and -1

Sol: $-2<-1$

$$\frac{-2}{1}\times\frac{6}{6}<\frac{-1}{1}\times\frac{6}{6}$$

$$\frac{-12}{6}<\frac{-6}{6}$$

$$\frac{-10}{5}<\frac{-9}{6}<\frac{-8}{6}<\frac{-7}{6}<\frac{-6}{6}<\frac{-5}{6}<\frac{-6}{6}$$

The five rational numbers between – 2 and – 1 are $\frac{-9}{6},\frac{-8}{6},\frac{-7}{6},\frac{-6}{6},\frac{-5}{6}$.

$$\Rightarrow\frac{-3}{2},\frac{-4}{3},\frac{-7}{6},-1,\frac{-5}{6}$$

(iii) $\frac{-4}{5}$ and $\frac{-2}{3}$

Sol: $\frac{-4}{5}<\frac{-2}{3}$

$$\frac{-4\times9}{5\times9}<\frac{-2\times15}{3\times15}$$

$$\frac{-36}{45}<\frac{-30}{45}$$

$$\frac{-36}{45}<\frac{-35}{45}<\frac{-34}{45}<\frac{-33}{45}<\frac{-32}{45}<\frac{-31}{45}<\frac{-30}{45}$$

The five rational numbers between $\frac{-4}{5}$ *and* $\frac{-2}{3}$ are $\frac{-35}{45}, \frac{-34}{45}, \frac{-33}{45}, \frac{-32}{45}, \frac{-31}{45}$

$\Rightarrow \frac{-7}{9}, \frac{-34}{45}, \frac{-11}{15}, \frac{-32}{45}, \frac{-31}{45}$

(iv) $\frac{-1}{2}$ **and** $\frac{2}{3}$

Sol: $\frac{-1}{2} < \frac{2}{3}$

$\frac{-1 \times 3}{2 \times 3} < \frac{2 \times 2}{3 \times 2}$

$\frac{-3}{6} < \frac{4}{6}$

$\frac{-3}{6} < \frac{-2}{6} < \frac{-1}{6} < \frac{0}{6} < \frac{1}{6} < \frac{2}{6} < \frac{3}{6} < \frac{4}{6}$

The five rational numbers between $\frac{-1}{2}$ *and* $\frac{2}{3}$ are $\frac{-2}{6}, \frac{-1}{6}, \frac{0}{6}, \frac{1}{6}, \frac{2}{6}$

$\Rightarrow \frac{-1}{3}, \frac{-1}{6}, 0, \frac{1}{6}, \frac{1}{3}$

2. **Write four more rational numbers in each of the following patterns:**

(i) $\frac{-3}{5}, \frac{-6}{10}, \frac{-9}{15}, \frac{-12}{20}, \ldots$

Sol: $\frac{-3}{5}, \quad \frac{-3 \times 2}{5 \times 2} = \frac{-6}{10}, \quad \frac{-3 \times 3}{5 \times 3} = \frac{-9}{15}, \quad \frac{-3 \times 4}{5 \times 4} = \frac{-12}{20}, \ldots$

The other numbers are

$\frac{-3 \times 5}{5 \times 5}, \quad \frac{-3 \times 6}{5 \times 6}, \quad \frac{-3 \times 7}{5 \times 7}, \quad \frac{-3 \times 8}{5 \times 8}$

$\Rightarrow \frac{-15}{25}, \quad \frac{-18}{30}, \quad \frac{-21}{35}, \quad \frac{-24}{40}$

(ii) $\frac{-1}{4}, \frac{-2}{8}, \frac{-3}{12}, ..$

Sol: $\frac{-1}{4}, \quad \frac{-1 \times 2}{4 \times 2} = \frac{-2}{8}, \quad \frac{-1 \times 3}{4 \times 3} = \frac{-3}{12}, ..$

The other numbers are

$\frac{-1 \times 4}{4 \times 4}, \quad \frac{-1 \times 5}{4 \times 5}, \quad \frac{-1 \times 6}{4 \times 6}, \quad \frac{-1 \times 7}{4 \times 7}$

$\Rightarrow \frac{-4}{16}, \quad \frac{-5}{20}, \quad \frac{-6}{24}, \quad \frac{-7}{28}$

(iii) $\frac{-1}{6}, \frac{2}{-12}, \frac{3}{18}, \frac{4}{-24}, ..$

Sol: $\frac{-1}{6}, \frac{2}{-12}, \frac{3}{-18}, \frac{4}{-24}, \ldots$

$\frac{-1}{6}, \frac{-1\times(-2)}{6\times(-2)} = \frac{2}{-12}, \frac{-1\times(-3)}{6\times(-3)} = \frac{3}{-18}, \frac{-1\times(-4)}{6\times(-4)} = \frac{4}{-24}, \ldots$

The other numbers are

$\frac{-1\times(-5)}{6\times(-5)}, \frac{-1\times(-6)}{6\times(-6)}, \frac{-1\times(-7)}{6\times(-7)}, \frac{-1\times(-8)}{6\times(-8)}$

$= \frac{5}{-30}, \frac{6}{-36}, \frac{7}{-42}, \frac{8}{-48},$

(iv) $\frac{-2}{3}, \frac{2}{-3}, \frac{4}{-6}, \frac{6}{-9}, \ldots$

Sol: $\frac{-2}{3}, \frac{-2\times(-1)}{3\times(-1)} = \frac{2}{-3}, \frac{-2\times(-2)}{3\times(-2)} = \frac{4}{-6}, \frac{-2\times(-3)}{3\times(-3)} = \frac{6}{-9}, \ldots$

The other numbers are

$\frac{-2\times(-4)}{3\times(-4)}, \frac{-2\times(-5)}{3\times(-5)}, \frac{-2\times(-6)}{3\times(-6)}, \frac{-2\times(-7)}{3\times(-7)}, \ldots$

$\frac{8}{-12}, \frac{10}{-15}, \frac{12}{-18}, \frac{14}{-21}$

If the numerator and denominator of a rational number are multiplied or divided by a non-zero integer, we get a rational number which is said to be equivalent to the given rational number.

3. Give four rational numbers equivalent to:

(i) $\frac{-2}{7}$

Sol: $\frac{-2}{7} = \frac{-2\times2}{7\times2} = \frac{-2\times3}{7\times3} = \frac{-2\times4}{7\times4} = \frac{-2\times5}{7\times5}$

$\frac{-2}{7} = \frac{-4}{14} = \frac{-6}{21} = \frac{-8}{28} = \frac{-10}{35}$

(ii) $\frac{5}{-3}$

Sol: $\frac{5}{-3} = \frac{5\times2}{-3\times2} = \frac{5\times3}{-3\times3} = \frac{5\times4}{-3\times4} = \frac{5\times5}{-3\times5}$

$$\frac{5}{-3}=\frac{10}{-6}=\frac{15}{-9}=\frac{20}{-12}=\frac{25}{-15}$$

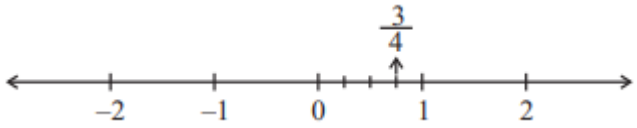
(iii) $\frac{4}{9}$

Sol: $\frac{4}{9}=\frac{4\times 2}{9\times 2}=\frac{4\times 3}{9\times 3}=\frac{4\times 4}{9\times 4}=\frac{4\times 5}{9\times 5}$

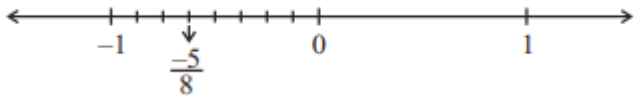
$$\frac{4}{9}=\frac{8}{18}=\frac{12}{27}=\frac{16}{36}=\frac{20}{45}$$

4. Draw the number line and represent the following rational numbers on it:

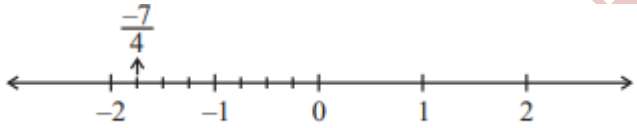
(i) $\frac{3}{4}$



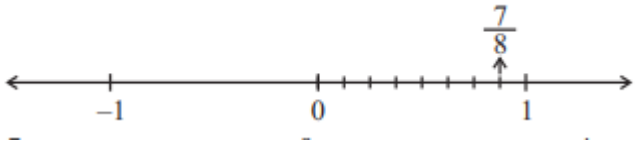
(ii) $\frac{-5}{8}$



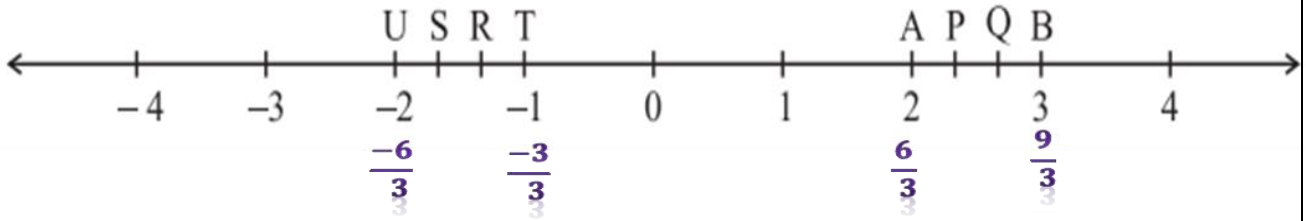
(iii) $\frac{-7}{4}$



(iv) $\frac{7}{8}$



5. The points P, Q, R, S, T, U, A and B on the number line are such that, TR = RS = SU and AP = PQ = QB. Name the rational numbers represented by P, Q, R and S.



Sol: Each one unit divided into three equal parts.

$-1=\frac{-3}{3}$, $-2=\frac{-6}{3}$, $2=\frac{6}{3}$, $3=\frac{9}{3}$

The rational numbers represented by P, Q, R and S are $\frac{7}{3}, \frac{8}{3}, \frac{-4}{3}, and\ \frac{-5}{3}$

6. Which of the following pairs represent the same rational number?

(i) $\frac{-7}{21}$ **and** $\frac{3}{9}$

Sol: $\frac{-7}{21}$ is negative and $\frac{3}{9}$ is positive .

So, $\frac{-7}{21}\ and\ \frac{3}{9}$ does not represent the same rational number.

(ii) $\frac{-16}{20}$ **and** $\frac{20}{-25}$

Sol: $\frac{-16}{20} = \frac{-16 \div 4}{20 \div 4} = \frac{-4}{5}$

$\frac{20}{-25} = \frac{20 \div (-5)}{-25 \div (-5)} = \frac{-4}{5}$

So, $\frac{-16}{20}$ and $\frac{20}{-25}$ represent the same rational number.

$\frac{-16}{20}$ *cm* $\frac{20}{-25}$

$(-16) \times (-25) = 400$

$(20) \times (20) = 400$

$\frac{-16}{20} = \frac{20}{-25}$

(iii) $\frac{-2}{-3}$ **and** $\frac{2}{3}$

Sol: $\frac{-2}{-3} = \frac{-2 \div (-1)}{-3 \div (-1)} = \frac{2}{3}$

So, $\frac{-2}{-3}$ and $\frac{2}{3}$ represent the same rational number.

(iv) $\frac{-3}{5}$ **and** $\frac{-12}{20}$

Sol: $\frac{-12}{20} = \frac{-12 \div 4}{20 \div 4} = \frac{-3}{5}$

So, $\frac{-3}{5}$ and $\frac{-12}{20}$ represent the same rational number.

(v) $\frac{8}{-5}$ **and** $\frac{-24}{15}$

Sol: $\frac{-24}{15} = \frac{-24 \div (-3)}{15 \div (-3)} = \frac{8}{-5}$

So, $\frac{8}{-5}$ and $\frac{-24}{15}$ represent the same rational number.

(vi) $\frac{1}{3}$ **and** $\frac{-1}{9}$

Sol: $\frac{1}{3}$ is positive and $\frac{-1}{9}$ is negative .

So, $\frac{1}{3}$ and $\frac{-1}{9}$ does not represent the same rational number.

(vii) $\frac{-5}{-9}$ **and** $\frac{5}{-9}$

Sol: $\frac{-5}{-9}$ is positive and $\frac{5}{-9}$ is negative .

So, $\frac{-5}{-9}$ and $\frac{5}{-9}$ does not represent the same rational number.

7. Rewrite the following rational numbers in the simplest form:

(i) $\frac{-8}{6} = \frac{-8 \div 2}{6 \div 2} = \frac{-4}{3}$

(ii) $\frac{25}{45} = \frac{25 \div 5}{45 \div 5} = \frac{5}{9}$

(iii) $\frac{-44}{72} = \frac{-44 \div 4}{72 \div 4} = \frac{-11}{18}$

(iv) $\frac{-8}{10} = \frac{-8 \div 2}{10 \div 2} = \frac{-4}{5}$

8. Fill in the boxes with the correct symbol out of >,< and =.

(*i*) $\frac{-5}{7} \boxed{<} \frac{2}{3}$

Sol: $\frac{-5}{7} \ldots\ldots\ldots \frac{2}{3}$

$\frac{-5 \times 3}{7 \times 3} \ldots\ldots\ldots \frac{2 \times 7}{3 \times 7}$

$\frac{-15}{21} < \frac{14}{21}$

(Negative number always less than positive number)

(ii) $\frac{-4}{5} \boxed{<} \frac{-5}{7}$

Sol: $\frac{-4}{5} \_\_\_\_ \frac{-5}{7}$

$\frac{-4 \times 7}{5 \times 7} \_\_\_\_ \frac{-5 \times 5}{7 \times 5}$

$\frac{-28}{35} < \frac{-25}{35}$

(iii) $\frac{-7}{8} \boxed{=} \frac{14}{-16}$

Sol: $\frac{-7}{8} \_\_\_\_ \frac{14}{-16}$

$\frac{-7 \times (-2)}{8 \times (-2)} \_\_\_\_ \frac{14}{-16}$

$\frac{14}{-16} = \frac{14}{-16}$

(iv) $\frac{-8}{5} \boxed{>} \frac{-7}{4}$

Sol: $\frac{-8}{5} \_\_\_\_ \frac{-7}{4}$

$\frac{-8 \times 4}{5 \times 4} \_\_\_\_ \frac{-7 \times 5}{4 \times 5}$

$\frac{-32}{20} \boxed{>} \frac{-35}{20}$

(v) $\frac{1}{-3} \boxed{<} \frac{-1}{4}$

Sol: $\frac{1}{-3}\_\_\_\frac{-1}{4}$

$\frac{1\times(-4)}{-3\times(-4)}\_\_\_\_\frac{-1\times 3}{4\times 3}$

$\frac{-4}{12}<\frac{-3}{12}$

$(vi)\ \frac{5}{-11}\boxed{=}\frac{-5}{11}$

$Sol:\ \frac{5}{-11}\_\_\_\_\frac{-5}{11}$

$\frac{5\times(-1)}{-11\times(-1)}\_\_\_\_\frac{-5}{11}$

$\frac{-5}{11}=\frac{-5}{11}$

(vii)0 $\boxed{>}\ \frac{-7}{6}$

Sol: 0 always greater than negative.

9. Which is greater in each of the following:

(i) $\frac{2}{3},\frac{5}{2}$

Sol: LCM of $3,2=6$

$\frac{2}{3}=\frac{2\times 2}{3\times 2}=\frac{4}{6}$

$\frac{5}{2}=\frac{5\times 3}{2\times 3}=\frac{15}{6}$

$\frac{4}{6}<\frac{15}{6}\Rightarrow\frac{2}{3}<\frac{5}{2}$

So, $\frac{5}{2}$ is greater.

k(ii) $\frac{-5}{6},\frac{-4}{3}$

Sol: LCM of $6,3=6$

$\frac{-5}{6}=\frac{-5\times 1}{6\times 1}=\frac{-5}{6}$

$\frac{-4}{3}=\frac{-4\times 2}{3\times 2}=\frac{-8}{6}$

$\frac{-8}{6}<\frac{-5}{6}\Rightarrow\frac{-4}{3}<\frac{-5}{6}$

So, $\frac{-5}{6}$ is greater.

(iii) $\frac{-3}{4},\frac{2}{-3}$

Sol: LCM of $4,3=12$

$\frac{-3}{4}=\frac{-3\times 3}{4\times 3}=\frac{-9}{12}$

$\frac{2}{-3}=\frac{2\times(-4)}{-3\times(-4)}=\frac{-8}{12}$

$\frac{-9}{12}<\frac{-8}{12}\Rightarrow\frac{-3}{4}<\frac{2}{-3}$

So, $\frac{2}{-3}$ is greater.

(iv) $\frac{-1}{4},\frac{1}{4}$

Sol: Every positive is greater than negative.

So, $\frac{1}{4}$ is greater.

(v) $-3\frac{2}{7},-3\frac{4}{5}$

Sol: LCM of $7,5=35$

$-3\frac{2}{7}=-\frac{23}{7}=-\frac{23\times 5}{7\times 5}=\frac{-115}{35}$

$-3\frac{4}{5}=\frac{-19}{5}=\frac{-19\times 7}{5\times 7}=\frac{-133}{35}$

$\frac{-133}{35}<\frac{-115}{35}\Rightarrow -3\frac{4}{5}<-3\frac{2}{7}$

So, $-3\frac{2}{7}$ is greater.

10. Write the following rational numbers in ascending order:

If we write rational numbers in ascending order then we convert all into like rational numbers.

(i) $\frac{-3}{5}, \frac{-2}{5}, \frac{-1}{5}$

Sol: Ascending order is, $\frac{-3}{5} < \frac{-2}{5} < \frac{-1}{5}$

(ii) $\frac{-1}{3}, \frac{-2}{9}, \frac{-4}{3}$

Sol: LCM of $3,9 = 9$

$$\frac{-1}{3} = \frac{-1 \times 3}{3 \times 3} = \frac{-3}{9}$$

$$\frac{-2}{9} = \frac{-2 \times 1}{9 \times 1} = \frac{-2}{9}$$

$$\frac{-4}{3} = \frac{-4 \times 3}{3 \times 3} = \frac{-12}{9}$$

Ascending order

$$\frac{-12}{9} < \frac{-3}{9} < \frac{-2}{9}$$

$$\therefore \frac{-4}{3} < \frac{-1}{3} < \frac{-2}{9}$$

(iii) $\frac{-3}{7}, \frac{-3}{2}, \frac{-3}{4}$

Sol: LCM of $7,2,4 = 28$

$$\frac{-3}{7} = \frac{-3 \times 4}{7 \times 4} = \frac{-12}{28}$$

$$\frac{-3}{2} = \frac{-3 \times 14}{2 \times 14} = \frac{-42}{28}$$

$$\frac{-3}{4} = \frac{-3 \times 7}{4 \times 7} = \frac{-21}{28}$$

Ascending order

$$\frac{-42}{28} < \frac{-21}{28} < \frac{-12}{28}$$

$$\therefore \frac{-3}{2} < \frac{-3}{4} < \frac{-3}{7}$$

OPERATIONS ON RATIONAL NUMBERS:

Addition

Adding rational numbers with same denominators, we add the numerators keeping the denominators same.

(i) $\frac{7}{3}+\left(\frac{-5}{3}\right)=\frac{7}{3}+\frac{(-5)}{3}=\frac{7+(-5)}{3}=\frac{2}{3}$

(ii) $\frac{6}{5}+\frac{(-2)}{5}=\frac{6+(-2)}{5}=\frac{4}{5}$

(iii) $\frac{3}{7}+\frac{(-5)}{7}=\frac{3+(-5)}{7}=\frac{-2}{7}$

(iv) $\frac{-7}{8}+\frac{5}{8}=\frac{-7+5}{8}=\frac{-2}{8}=\frac{-1}{4}$

(v) $\frac{-13}{7}+\frac{6}{7}=\frac{-13+6}{7}=\frac{7}{7}=1$

(vi) $\frac{19}{5}+\left(\frac{-7}{5}\right)=\frac{19+(-7)}{5}=\frac{12}{5}$

If we add rational numbers with different denominators. Then, we find the equivalent rational numbers of the given rational numbers with their LCM as the denominator and add the rational numbers.

(i) $\frac{-7}{5}+\frac{-2}{3}=\frac{-7\times 3}{5\times 3}+\frac{-2\times 5}{3\times 5}=\frac{-21}{15}+\frac{-10}{15}=\frac{(-21)+(-10)}{15}=\frac{-31}{15}$

(ii) $\frac{-3}{7}+\frac{2}{3}=\frac{-3\times 3}{7\times 3}+\frac{2\times 7}{3\times 7}=\frac{-9}{21}+\frac{14}{21}=\frac{-9+14}{21}=\frac{5}{21}$

(iii) $\frac{-5}{6}+\frac{-3}{11}=\frac{-5\times 11}{6\times 11}+\frac{-3\times 6}{11\times 6}=\frac{-55}{66}+\frac{-18}{66}=\frac{-55+(-18)}{66}=\frac{-73}{66}$

Additive Inverse

$\frac{a}{b}+\left(\frac{-a}{b}\right)=\frac{a+(-a)}{b}=\frac{0}{b}=0$

The additive inverse of $\frac{a}{b}=\frac{-a}{b}$

The additive inverse of $\frac{-a}{b}=\frac{a}{b}$

(i) **The additive inverse of** $\frac{-3}{9}=\frac{3}{9}$

(ii) **The additive inverse of** $\frac{-9}{11}=\frac{9}{11}$

(iii) **The additive inverse of** $\frac{5}{7}=\frac{-5}{7}$

Example 6: Satpal walks $\frac{2}{3}$km from a place P, towards east and then from there $1\frac{5}{7}$ km towards west. Where will he be now from P?

Sol: $\frac{2}{3}+\left(-1\frac{5}{7}\right)=\frac{2}{3}+\frac{(-12)}{7}=\frac{2\times 7}{3\times 7}+\frac{(-12)\times 3}{7\times 3}$

$=\frac{14}{21}+\frac{(-36)}{21}=\frac{14+(-36)}{21}=\frac{-22}{21}=-1\frac{1}{21}$

Subtraction

While subtracting two rational numbers, we add the additive inverse of the rational number that is being subtracted, to the other rational number.

$$\frac{a}{b}-\frac{c}{q}=\frac{a\times q-c\times b}{b\times q}$$

$(i)\ \frac{5}{7}-\frac{3}{8}=\frac{5\times 8-3\times 7}{7\times 8}=\frac{40-21}{56}=\frac{19}{56}$

$(ii)\ \frac{7}{8}-\frac{5}{9}=\frac{7\times 9-5\times 8}{8\times 9}=\frac{63-40}{72}=\frac{23}{72}$

$(iii)\ \frac{3}{11}-\frac{8}{7}=\frac{3\times 7-8\times 11}{11\times 7}=\frac{21-88}{77}=\frac{-67}{77}$

$(iv)\ \frac{7}{9}-\frac{2}{5}=\frac{7\times 5-2\times 9}{9\times 5}=\frac{35-18}{45}=\frac{17}{45}$

$(v)\ 1\frac{2}{3}-2\frac{4}{5}=\frac{5}{3}-\frac{14}{5}=\frac{5\times 5-14\times 3}{3\times 5}=\frac{25-42}{15}=\frac{-17}{15}$

$(vi)\ \frac{2}{7}-\left(\frac{-5}{6}\right)=\frac{2}{7}+\frac{5}{6}=\frac{2\times 6+5\times 7}{7\times 6}=\frac{12+35}{56}=\frac{47}{56}$

$(vii)\ 2\frac{1}{5}-\frac{(-1)}{3}=\frac{11}{5}+\frac{1}{3}=\frac{11\times 3+1\times 5}{5\times 3}=\frac{33+5}{15}=\frac{38}{15}$

Multiplication

While multiplying a rational number by an integer, we multiply the numerator by that integer, keeping the denominator unchanged.

$(i)\ \frac{-2}{9}\times(-5)=\frac{-2\times(-5)}{9}=\frac{10}{9}=1\frac{1}{9}$

$(ii)\ \frac{3}{11}\times(-2)=\frac{3\times(-2)}{11}=\frac{-6}{11}$

$(iii)\ \frac{-3}{5}\times 7=\frac{-3\times 7}{5}=\frac{-21}{5}$

$(iv)\ \frac{-6}{5}\times(-2)=\frac{-6\times(-2)}{5}=\frac{12}{5}$

To multiply two rational numbers, we multiply their numerators and denominators separately, and write theproduct as $\frac{\text{productof numerators}}{\text{product of denominators}}$

$(i)\ \frac{-3}{8}\times\frac{5}{7}=\frac{-3\times 5}{8\times 7}=\frac{-15}{56}$

$(ii)\ \frac{-5}{8}\times\frac{-9}{7}=\frac{-5\times(-9)}{8\times 7}=\frac{45}{56}$

(iii) $\frac{-3}{4} \times \frac{1}{7} = \frac{-3 \times 1}{4 \times 7} = \frac{-3}{28}$

(iv) $\frac{2}{3} \times \frac{-5}{9} = \frac{2 \times (-5)}{3 \times 9} = \frac{-10}{27}$

Division

Reciprocal:(Multiplicative inverse)

If product of two rational numbers is 1 then they are said to be reciprocals of each other

Reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$

(i) **The reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$**

(ii) **The reciprocal of $\frac{-6}{11}$ is $\frac{-11}{6}$**

(iii) **The reciprocal of $\frac{-8}{5}$ is $\frac{-5}{8}$**

To divide one rational number by the other non-zero rational number we multiply the rational number by the reciprocal of the other.

(i) $\frac{4}{9} \div \frac{-5}{7} = \frac{4}{9} \times \frac{-7}{5} = \frac{4 \times (-7)}{9 \times 5} = \frac{-28}{45}$

(ii) $\frac{-6}{5} \div \frac{-2}{3} = \frac{-6}{5} \times \frac{-3}{2} = \frac{(-6) \times (-3)}{5 \times 2} = \frac{18}{10} = \frac{9}{5}$

(iii) $\frac{2}{3} \div \frac{-7}{8} = \frac{2}{3} \times \frac{-8}{7} = \frac{2 \times (-8)}{3 \times 7} = \frac{-16}{21}$

(iv) $\frac{-6}{7} \div \frac{5}{7} = \frac{-6}{7} \times \frac{7}{5} = \frac{(-6) \times \cancel{7}}{\cancel{7} \times 5} = \frac{-6}{5}$

Exercise 8.2

1. **Find the sum:**

(i) $\frac{5}{4} + \frac{-11}{4} = \frac{5 + (-11)}{4} = \frac{-6}{4} = \frac{-3}{2}$

(ii) $\frac{5}{3} + \frac{3}{5} = \frac{5 \times 5}{3 \times 5} + \frac{3 \times 3}{5 \times 3} = \frac{25}{35} + \frac{9}{35}$

$= \frac{25 + 9}{35} = \frac{34}{35}$

(iii) $\frac{-9}{10} + \frac{22}{15} = \frac{-9 \times 3}{10 \times 3} + \frac{22 \times 2}{15 \times 2} = \frac{-27}{30} + \frac{44}{30}$

$= \frac{-27 + 44}{30} = \frac{17}{30}$

(iv) $\frac{-3}{-11}+\frac{5}{9}=\frac{-3\times(-9)}{-11\times(-9)}+\frac{5\times 11}{9\times 11}$

$=\frac{27}{99}+\frac{55}{99}=\frac{27+55}{99}=\frac{82}{99}$

(v) $\frac{-8}{19}+\frac{(-2)}{57}=\frac{-8\times 3}{19\times 3}+\frac{(-2)\times 1}{57\times 1}$

$=\frac{-24}{57}+\frac{-2}{57}=\frac{-24+(-2)}{57}=\frac{-26}{57}$

(vi) $\frac{-2}{3}+0=\frac{-2}{3}$

(vii) $-2\frac{1}{3}+4\frac{3}{5}=\frac{-7}{3}+\frac{23}{5}=\frac{-7\times 5}{3\times 5}+\frac{23\times 3}{5\times 3}$

$=\frac{-35}{15}+\frac{69}{15}=\frac{-35+69}{15}=\frac{34}{15}$

2. Find

(i) $\frac{7}{24}-\frac{17}{36}=\frac{7\times 3}{24\times 3}-\frac{17\times 2}{36\times 2}=\frac{21}{72}-\frac{34}{72}$

$=\frac{21-34}{72}=\frac{-13}{72}$

(ii) $\frac{5}{63}-\left(\frac{-6}{21}\right)=\frac{5}{63}+\frac{6}{21}=\frac{5}{63}+\frac{6\times 3}{21\times 3}$

$=\frac{5}{63}+\frac{18}{63}=\frac{23}{63}$

(iii) $\frac{-6}{13}-\left(\frac{-7}{15}\right)=\frac{-6}{13}+\frac{7}{15}=\frac{-6\times 15}{13\times 15}+\frac{7\times 13}{15\times 13}$

$=\frac{-90}{195}+\frac{91}{195}=\frac{1}{195}$

(iv) $\frac{-3}{8}-\frac{7}{11}=\frac{-3\times 11}{8\times 11}-\frac{7\times 8}{11\times 8}=\frac{-33}{88}-\frac{56}{88}$

$=\frac{-33-56}{88}=\frac{-89}{88}$

(v) $-2\frac{1}{9}-6=\frac{-19}{9}-\frac{6}{1}=\frac{-19}{9}-\frac{6\times 9}{1\times 9}=\frac{-19}{9}-\frac{54}{9}$

$=\frac{-19-54}{9}=\frac{-73}{9}$

3. Find the product:

(i) $\frac{9}{2} \times \left(\frac{-7}{4}\right) = \frac{9 \times (-7)}{2 \times 4} = \frac{-63}{8}$

(ii) $\frac{3}{10} \times (-9) = \frac{3 \times (-9)}{10} = \frac{-27}{10}$

(iii) $\frac{-6}{5} \times \frac{9}{11} = \frac{-6 \times 9}{5 \times 11} = \frac{-54}{55}$

(iv) $\frac{3}{7} \times \left(\frac{-2}{5}\right) = \frac{3 \times (-2)}{7 \times 5} = \frac{-6}{35}$

(v) $\frac{3}{11} \times \frac{2}{5} = \frac{3 \times 2}{11 \times 5} = \frac{6}{55}$

(vi) $\frac{3}{-5} \times \frac{-5}{3} = \frac{3 \times (-5)}{-5 \times 3} = \frac{-15}{-15} = 1$

4. Find the value of:

(i) $(-4) \div \frac{2}{3} = \frac{-4}{1} \times \frac{3}{2} = \frac{-4 \times 3}{1 \times 2}$

$= \frac{-12}{2} = -6$

(ii) $\frac{-3}{5} \div 2 = \frac{-3}{5} \times \frac{1}{2} = \frac{-3}{10}$

(iii) $\frac{-4}{5} \div (-3) = \frac{-4}{5} \times \frac{1}{-3} = \frac{-4}{-15}$

$= \frac{4}{15}$

(iv) $\frac{-1}{8} \div \frac{3}{4} = \frac{-1}{8} \times \frac{4}{3} = \frac{-4}{24} = \frac{-1}{6}$

(v) $\frac{-2}{13} \div \frac{1}{7} = \frac{-2}{13} \times \frac{7}{1} = \frac{-14}{13}$

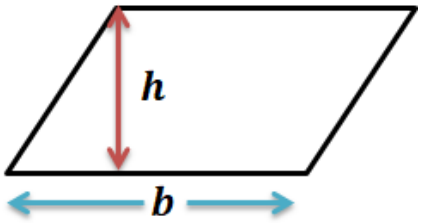
(vi) $\frac{-7}{12} \div \left(\frac{-2}{13}\right) = \frac{-7}{12} \times \frac{-13}{2} = \frac{91}{24}$

(vii) $\frac{3}{13} \div \left(\frac{-4}{65}\right) = \frac{3}{13} \times \frac{-65}{4} = \frac{3 \times (-5)}{4}$

$= \frac{-15}{4}$

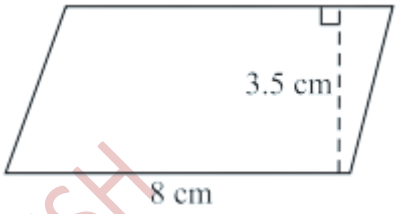
1. **Area of parallelogram** = base × height = b × h.

Any side of a parallelogram can be chosen as base(b) of the parallelogram. The perpendicular dropped on that side from the opposite vertex is known as height (h)(altitude).



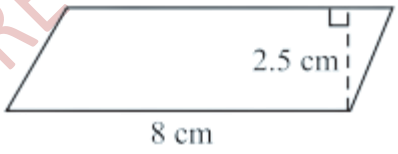
TRY THESE

Find the area of following parallelograms:



(i) Area of parallelogram = base × height

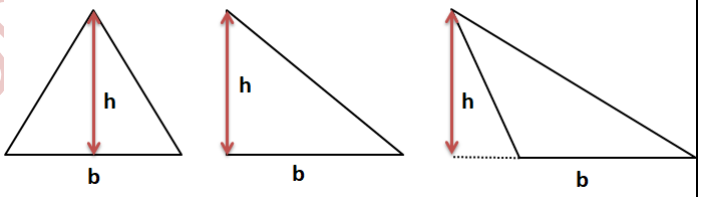
= b × h=8 cm×3.5 cm=28 cm²



(ii) Area of parallelogram = base × height

= b × h=8 cm×2.5 cm=20 cm²

AREA OF A TRIANGLE



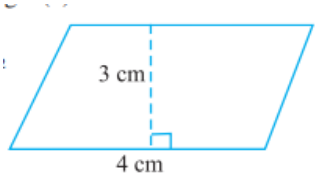
Area of each triangle

$$= \frac{1}{2} \times (\text{base} \times \text{height})$$

$$= \frac{1}{2} \times b \times h$$

All the congruent triangles are equal in area but the triangles equal in area need not be congruent.

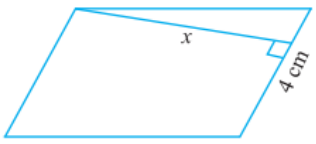
Example 1: One of the sides and the corresponding height of a parallelogram are 4 cm and 3 cm respectively. Find the area of the parallelogram (Fig 9.8).



Sol: Base (b) = 4 cm, height (h) = 3 cm

Area of the parallelogram = b × h= 4 cm × 3 cm = 12 cm²

Example 2: Find the height 'x' if the area of the parallelogram is 24 cm² and the base is 4 cm



Sol: Base (b) = 4 cm, height (h) = x cm

The area of the parallelogram = 24 cm²

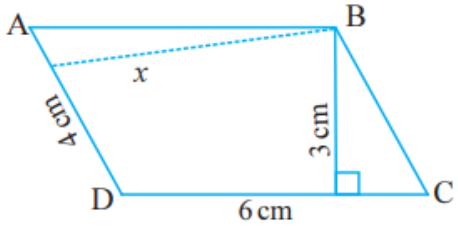
$b \times h = 24$

$4 \times x = 24$

$x = \frac{24}{4} = 6\ cm$

$\therefore$ The height of the parallelogram is 6 cm.

Example 3: The two sides of the parallelogram ABCD are 6 cm and 4 cm. The height corresponding to the base CD is 3 cm (Fig 9.10). Find the (i) area of the parallelogram. (ii) the height corresponding to the base AD.



Sol: (i) Area of parallelogram = b × h = 6 cm × 3 cm = 18 cm^2

(ii) Base (b) = 4 cm, height = x (say),

Area = 18 cm^2

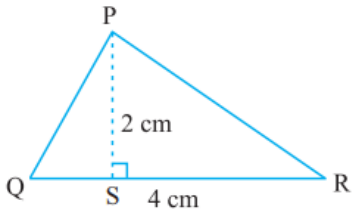
b × h = 18

$4 \times x = 18$

$x = \frac{18}{4} = 4.5\ cm$

Thus, the height corresponding to base AD is 4.5 cm.

Example 4: Find the area of the following triangles

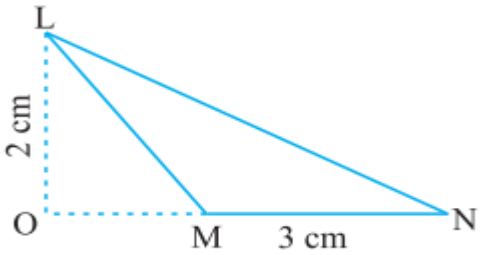


(i) Base (b) = 4 cm, height = 2 cm

Area of each triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 4\ cm \times 2\ cm = 4\ cm^2$

(ii) Base (b) = 3 cm, height = 2 cm

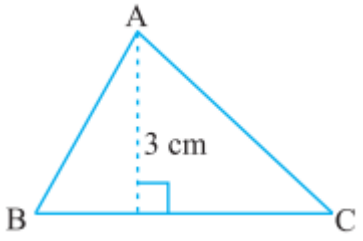


Area of each triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 3\ cm \times 2\ cm = 3\ cm^2$

Example 5: Find BC, if the area of the triangle ABC is 36 cm^2 and the height AD is 3 cm

Sol: Height (h) = 3 cm



Area = 36 cm^2

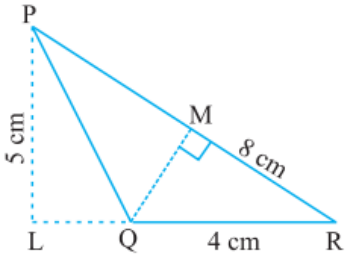
$\frac{1}{2} \times b \times h = 36$

$\frac{1}{2} \times b \times 3 = 36$

$$b = \frac{36 \times 2}{3} = 12 \times 2 = 24\ cm$$

BC $= 24\ cm$

Example 6: In ΔPQR, PR = 8 cm, QR = 4 cm and PL = 5 cm (Fig 9.13). Find: (i) the area of the ΔPQR (ii) QM



Sol: (i) QR = base(b) =4 cm, PL = height (h)= 5 cm

Area of the triangle PQR $= \frac{1}{2}$bh $= \frac{1}{2} \times 4$ cm $\times 5$ cm $= 10$ cm^2

(ii) Base=PR=8 cm, Height=QM=?

Area of the triangle PQR $= 10\ cm^2$

$$\frac{1}{2} \times b \times h = 10$$

$$\frac{1}{2} \times 8 \times h = 10$$

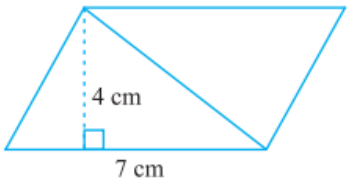
$$h = \frac{10 \times 2}{8} = 2.5\ cm$$

QM $= 2.5\ cm$

EXERCISE 9.1

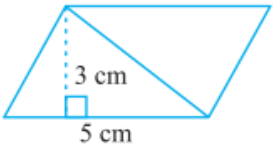
1. Find the area of each of the following parallelograms:

(a)



Area of the parallelogram $= b \times h$
$= 7$ cm $\times 4$ cm $= 28$ cm^2

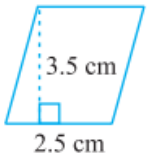
(b)



Area of the parallelogram $= b \times h$

$= 5$ cm $\times 3$ cm $= 15$ cm^2

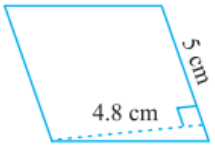
(c)



Area of the parallelogram $= b \times h$

$= 2.5$ cm $\times 3.5$ cm $= 8.75$ cm^2

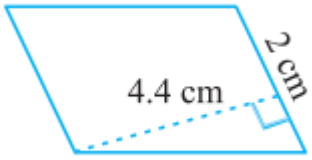
(d)



Area of the parallelogram $= b \times h$

$= 5$ cm $\times 4.8$ cm $= 24$ cm^2

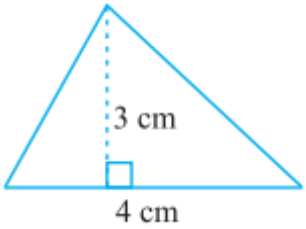
(e)



Area of the parallelogram $= b \times h = 2 \text{ cm} \times 4.4 \text{ cm} = 8.8 \text{ cm}^2$

2. Find the area of each of the following triangles:

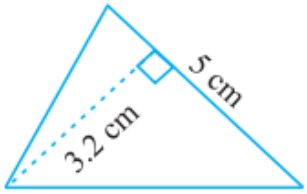
(a)



Area of the triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 4 \text{ cm} \times 3 \text{ cm} = 6 \text{ cm}^2$

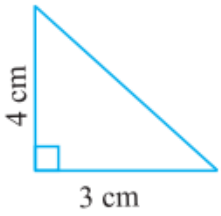
(b)



Area of the triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 5 \text{ cm} \times 3.2 \text{ cm} = 8 \text{ cm}^2$

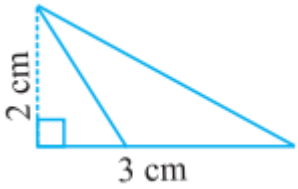
(c)



Area of the triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 3 \text{ cm} \times 4 \text{ cm} = 6 \text{ cm}^2$

(d)



Area of the triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 3 \text{ cm} \times 2 \text{ cm}$

$= 3 \text{ cm}^2$

3. Find the missing values:

Base = b, Height = h, Area of the Parallelogram = A

(a) $b = 20\ cm$, h =?, $A = 246\ cm^2$

Sol: b × h = A

$20 \times h = 246$

$h = \frac{246}{20} = 12.3$

Height=12.3 cm

(b) $\mathbf{b} = ?,\ \mathbf{h} = \mathbf{15\ cm},\ \mathbf{A} = \mathbf{154.5\ cm^2}$

Sol: $b \times h = A$

$b \times 15 = 154.5$

$b = \frac{154.5}{15} = 10.3$

Base=10.3 cm

(c) $\boldsymbol{b} = ?,\ \boldsymbol{h} = \mathbf{8.4\ cm},\ \boldsymbol{A} = \mathbf{48.72\ cm^2}$

$Sol: b \times h = A$

$b \times 8.4 = 48.72$

$b = \frac{48.72}{8.4} = \frac{487.2}{84} = 5.8$

Base=5.8 cm

(d) $\boldsymbol{b} = \mathbf{15.6\ cm},\ \boldsymbol{h} = ?,\ \boldsymbol{A} = \mathbf{16.38\ cm^2}$

$Sol: b \times h = A$

$15.36 \times h = 16.38$

$h = \frac{16.38}{15.36} = \frac{1638}{1536} = 1.06$

Height=1.06 cm

4. Find the missing values:

(i) $\mathbf{Base} = \mathbf{15\ cm}, \mathbf{Height} = ?, \mathbf{Area\ of\ Triangle} = \mathbf{87\ cm^2}$

Sol: $\frac{1}{2} \times$ Base $\times$ Height $=$ Area of the triangle

$\frac{1}{2} \times 15 \times$ Height $= 87$

Height $= \frac{87 \times 2}{15} = \frac{174}{5} = 11.6$ cm

$(\boldsymbol{ii})$ $\mathbf{Base} = ?, \mathbf{Height} = \mathbf{31.4\ cm}, \mathbf{Area\ of\ Triangle} = \mathbf{1256\ cm^2}$

Sol: $\frac{1}{2} \times$ Base $\times$ Height $=$ Area of the triangle

$\frac{1}{2} \times$ Base $\times 31.4 = 1256$

$$\text{Base} = \frac{1256 \times 2}{31.4} = \frac{2512}{31.4} = \frac{25120}{314} = 80\ cm$$

(iii) Base = 22 cm, Height =?, Area of Triangle = 170.5 cm^2

Sol: $\frac{1}{2} \times \text{Base} \times \text{Height} = \text{Area of the triangle}$

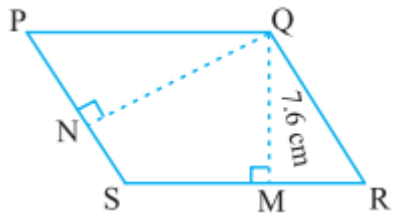
$$\frac{1}{2} \times 22 \times \text{Height} = 170.5$$

$$\text{Height} = \frac{170.5 \times 2}{22} = \frac{170.5}{11} = 15.5\ cm$$

5. PQRS is a parallelogram (Fig 9.14). QM is the height from Q to SR and QN is the height from Q to PS. If SR = 12 cm and QM = 7.6 cm. Find: (a) the area of the parallegram PQRS (b) QN, if PS = 8 cm

Sol: (a) The area of the parallelogram PQRS = Base × Height

$$= SR \times QM = 12 \times 7.6 = 91.2\ cm^2$$



(b) The area of the parallelogram PQRS=91.2 cm^2

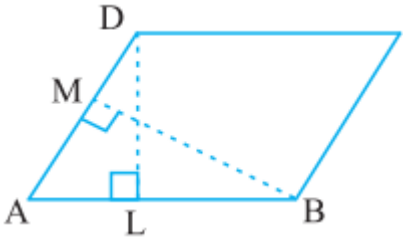
$$PS \times QN = 91.2$$

$$8 \times QN = 91.2$$

$$QN = \frac{91.2}{8} = 11.4\ cm$$

6. DL and BM are the heights on sides AB and AD respectively of parallelogram ABCD (Fig 9.15). If the area of the parallelogram is 1470 cm^2, AB = 35 cm and AD = 49 cm, find the length of BM and DL.

Sol: **The area of the parallelogram = 1470 cm^2**



Taking AD is base

$$AD \times BM = 1470$$

$$49 \times BM = 1470$$

$$BM = \frac{1470}{49} = 30\ \text{cm}$$

Taking AB is base

$$AB \times DL = 1470$$

$$35 \times DL = 1470$$

$$DL = \frac{1470}{35} = 42\ \text{cm}$$

7. ΔABC is right angled at A (Fig 9.16). AD is perpendicular to BC. If AB = 5 cm, BC = 13 cm and AC = 12 cm, Find the area of ΔABC. Also find the length of AD.

Sol: In ΔABC, $\angle A = 90^0$

Base 5cm and height=12 cm

Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 5 \times 12 = 30\ cm^2$

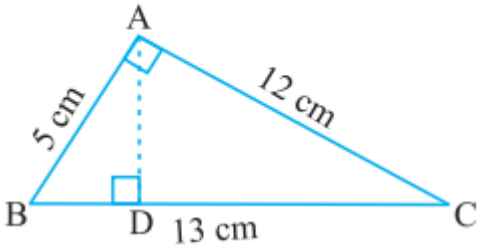
If take base=BD=13 cm then height=AD

Area of $\Delta ABC = 30cm^2$

$\frac{1}{2} \times BD \times AD = 30$

$\frac{1}{2} \times 13 \times AD = 30$

$AD = \frac{30 \times 2}{13} = \frac{60}{13} = 4.61\ cm$



8. ΔABC is isosceles with AB = AC = 7.5 cm and BC = 9 cm (Fig 9.17). The height AD from A to BC, is 6 cm. Find the area of ΔABC. What will be the height from C to AB i.e., CE?

Sol: If take base=BC=9 cm then height=AD=6 cm

Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 9 \times 6 = 27\ cm^2$

If take base=AB=7.5 cm then height=CE

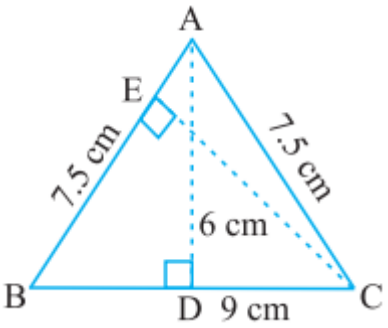
Area of $\Delta ABC = 27\ cm^2$

Area of $\Delta ABC = 27cm^2$

$\frac{1}{2} \times AB \times CE = 27$

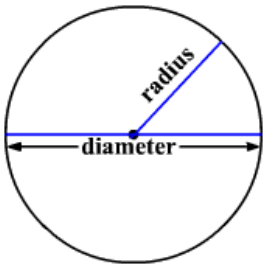
$\frac{1}{2} \times 7.5 \times CE = 27$

$CE = \frac{27 \times 2}{7.5} = \frac{54}{7.5} = \frac{540}{75} = \frac{108}{15} = \frac{36}{5} = 7.2\ cm$



CIRCLES

Circumference of a Circle: The distance around a circular region is known as its circumference.



The ratio of Circumference to Diameter is a constant and is denoted by π (pi). Its approximate value is $\frac{22}{7}$ or 3.14.

The diameter (d) of a circle is twice the radius (r) i.e., d = 2r

Circumference of a Circle= πd =2πr.

Example 7 : What is the circumference of a circle of diameter 10 cm (Take π = 3.14)?

Sol: Diameter of the circle (d) = 10 cm

Circumference of circle = πd = 3.14 × 10 cm = 31.4 cm

Example 8: What is the circumference of a circular disc of radius 14 cm? $\left(\textbf{Use } \pi = \frac{22}{7}\right)$

Sol: Radius of circular disc (r) = 14 cm

Circumference of disc $= 2\pi r = 2 \times \frac{22}{7} \times 14\ cm = 88\ cm$

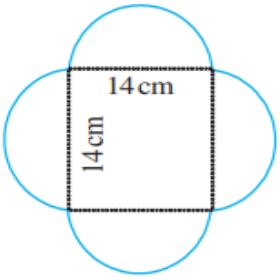
Example 9: The radius of a circular pipe is 10 cm. What length of a tape is required to wrap once around the pipe (π = 3.14)

Sol: Radius of the pipe (r) = 10 cm

Circumference of the pipe = 2πr== 2 × 3.14 × 10 cm== 62.8 cm

∴ Length of the tape needed to wrap once around the pipe is 62.8 cm.

Example 10: Find the perimeter of the given shape (Take $\pi = \frac{22}{7}$).



Sol: Circumference of the semicircle $= \frac{1}{2}\pi d = \frac{1}{2} \times \frac{22}{7} \times 14\ cm = 22\ cm.$

Therefore, perimeter of the given figure = 4 × 22 cm = 88 cm

Example 11: Sudhanshu divides a circular disc of radius 7 cm in two equal parts. What is the perimeter of each semicircular shape disc? (Use $\pi = \frac{22}{7}$).

Sol: Radius (r) = 7 cm

The circumference of circle = 2πr

The circumference of the semicircle $= \frac{1}{2} \times 2\pi r = \pi r = \frac{22}{7} \times 7$ cm = 22cm

The diameter of the circle = 2r = 2 × 7 cm = 14 cm

Perimeter of each semi-circular disc = 22 cm + 14 cm = 36 cm.

Area of Circle

$The\ area\ of\ the\ circle = \pi r^2 = \pi \times \frac{d^2}{4}$

Example 12: Find the area of a circle of radius 30 cm (use π = 3.14).

Sol: Radius(r) = 30 cm

Area of the circle $= \pi r^2 = 3.14 \times 30^2 = 3.14 \times 900 = 2{,}826\ cm^2$

Example13: Diameter of a circular garden is 9.8 m. Find its area.

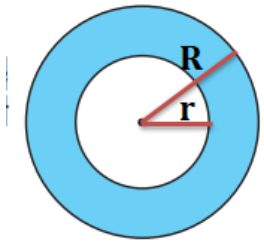
Sol: Diameter (d) = 9.8 m.

Radius (r) = 9.8 ÷ 2 = 4.9 m

Area of the circle $= \pi r^2 = \frac{22}{7} \times (4.9)^2 = \frac{22}{\cancel{7}} \times \cancel{4.9}^{0.7} \times 4.9 = 22 \times 0.7 \times 4.9 = 75.46\ m^2$

Example 14:The adjoining figure shows two circles with the same centre. The radius of the larger circle is 10 cm and the radius of the smaller circle is 4 cm.

Find: (a) the area of the larger circle (b) the area of the smaller circle (c) the shaded area between the two circles. (π = 3.14)



Sol: (a) Radius of the larger circle (R) = 10 cm

Area of the larger circle $= \pi R^2 = 3.14 \times 10 \times 10 = 3.14 \times 100$
$= 314\ cm^2$

(b) Radius of the smaller circle (r)= 4 cm

Area of the smaller circle $= \pi r^2 = 3.14 \times 4 \times 4 = 3.14 \times 16 = 50.24\ cm^2$

(c) Area of the shaded region = 314 – 50.24 = 263.76 cm^2

EXERCISE 9.2

1. Find the circumference of the circles with the following radius: (Take $\pi = \frac{22}{7}$).
(a) 14 cm (b) 28 mm (c) 21 cm

Sol: (a) Radius (r)=14 cm

Circumference of the circle $= 2\pi r$

$= 2 \times \frac{22}{7} \times 14\ cm = 44 \times 2\ cm = 88\ cm$

(b) Radius (r)=28 mm

Circumference of the circle $= 2\pi r$

$= 2 \times \frac{22}{7} \times 28\ mm = 44 \times 4\ mm = 176\ mm$

(c) Radius (r)=21 cm

Circumference of the circle $= 2\pi r$

$= 2 \times \frac{22}{7} \times 21\ cm = 44 \times 3\ cm = 132\ cm$

2. Find the area of the following circles, given that:
(a)radius = 14 mm $\left(\text{Take } \pi = \frac{22}{7}\right)$. (b)diameter = 49 m , (c) radius = 5 cm

Sol: (a)radius (r) = 14 mm

Area of the circle $= \pi r^2$

$= \frac{22}{7} \times 14\ mm \times 14\ mm = 22 \times 2 \times 14\ mm^2 = 616\ mm^2$

(b)diameter(d) = 49 m, radius (r) $= \frac{d}{2} = \frac{49}{2} = 24.5$ m.

Area of the circle $= \pi r^2$

$= \frac{22}{7} \times 24.5\ m \times 24.5\ m = 22 \times 3.5 \times 24.5\ m^2 = 1886.5\ m^2$

(c)radius (r) = 5 cm

Area of the circle $= \pi r^2$

$= \frac{22}{7} \times 5 \text{ cm} \times 5 \text{ cm} = \frac{550}{7}\ cm^2 = 78.57\ cm^2$

3. If the circumference of a circular sheet is 154 m, find its radius. Also find the area of the sheet. (Take $\pi = \frac{22}{7}$).

Sol: Circumference of the circular sheet =154 m

$2\pi r = 154$

$2 \times \frac{22}{7} \times r = 154$

$r = \frac{\cancel{154}^{\cancel{77}^{7}} \times 7}{\cancel{2} \times \cancel{22}_{2}} = \frac{49}{2} = 24.5\ m$

Area of the circular sheet $= \pi r^2 = \frac{22}{7} \times 24.5 \text{ m} \times 24.5 \text{ m}$

$= 22 \times 3.5 \times 24.5\ m^2 = 1886.5\ m^2$

4. A gardener wants to fence a circular garden of diameter 21m. Find the length of the rope he needs to purchase, if he makes 2 rounds of fence. Also find the cost of the rope, if it costs ` 4 per meter. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Sol: Diameter(d) = 21m, Radius(r) $= \frac{21}{2}$ m

Circumference of circular garden $= 2\pi r = \cancel{2} \times \frac{22}{\cancel{7}} \times \frac{\cancel{21}^{3}}{\cancel{2}} = 22 \times 3 = 66\ m$

The length of rope required for fencing=2×66 m=132 m

The cost of 1 m rope=₹ 4

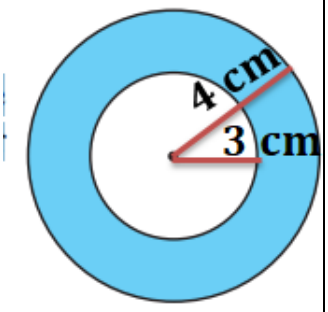
Total cost of the rope=₹ 4×132=₹ 528

5. From a circular sheet of radius 4 cm, a circle of radius 3 cm is removed. Find the area of the remaining sheet. (Take π = 3.14)

Sol: (a) Radius of the larger circle (R) = 4 cm

Area of the larger circle $= \pi R^2 = 3.14 \times 4 \times 4$

$= 3.14 \times 16 = 50.24\ cm^2$



(b) Radius of the smaller circle (r)= 3 cm

Area of the smaller circle $= \pi r^2 = 3.14 \times 3 \times 3$

$= 3.14 \times 9 = 28.26\ cm^2$

(c) Area of the remaining sheet =50.24– 28.16 =21.98cm²

6. Saima wants to put a lace on the edge of a circular table cover of diameter 1.5 m. Find the length of the lace required and also find its cost if one meter of the lace costs ` 15. (Take π = 3.14)

Sol: Diameter(d) $= 1.5 = \frac{15}{10} = \frac{3}{2}m$, radius(r) $= \frac{3}{2 \times 2} = \frac{3}{4}m$

Circumference of the table $= 2\pi r = \cancel{2} \times \cancel{3.14}^{1.57} \times \frac{3}{4_{\cancel{2}}}$

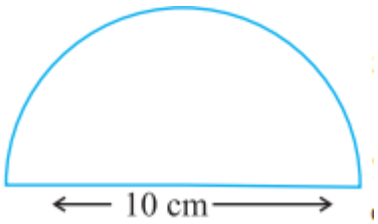
$= 1.57 \times 3 = 4.71\ m$

The cost of 1 m lace=₹ 15

Total cost of the lace=₹15×4.71=₹70.65

7. Find the perimeter of the adjoining figure, which is a semicircle including its diameter.

Sol: Diameter (d) = 10cm, Radius(r)=5 cm



The perimeter of the figure

$= Circumference\ of\ semi\ circle + diameter$

$= \pi r + d = \frac{22}{7} \times 5 + 10 = \frac{110}{7} + 10 = 15.7 + 10 = 25.7\ cm$

8. **Find the cost of polishing a circular table-top of diameter 1.6 m, if the rate of polishing is ₹ 15/m² (Take π = 3.14)**

Sol: Diameter $= 1.6\ m$, radius $= \frac{1.6}{2} = 0.8m$

Area of the table top $= \pi r^2 = 3.14 \times 0.8 \times 0.8 = 2.0096\ m^2$

The cost of 1 m² polishing= ₹15×2.0096=₹30.14

9. Shazli took a wire of length 44 cm and bent it into the shape of a circle. Find the radius of that circle. Also find its area. If the same wire is bent into the shape of a square, what will be the length of each of its sides? Which figure encloses more area, the circle or the square?

$\left(\textbf{Take } \pi = \frac{22}{7}\right)$

Sol: Length of wire=44 cm

Circumference of the circle=44 cm

$2\pi r = 44$

$2 \times \frac{22}{7} \times r = 44$

$r = \frac{44 \times 7}{2 \times 22} = 7$ cm

Area of the circle $= \pi r^2 = \frac{22}{\cancel{7}} \times \cancel{7} \times 7 = 22 \times 7 = 154\ cm^2$

Perimeter of the square=44 cm

4×side=44

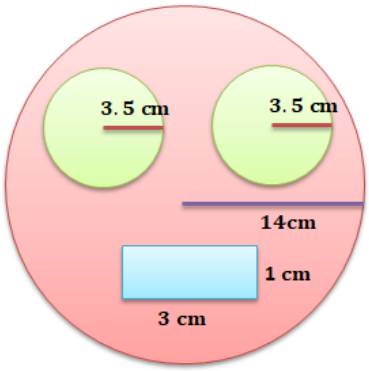
Side $= \frac{44}{4} = 11$cm

Area of the square = side × side = $11 \times 11 = 121\text{cm}^2$

Area of the circle = 154 cm^2 and Area of the square = 121cm^2

The area of circle is greater than that of square.

10. From a circular card sheet of radius 14 cm, two circles of radius 3.5 cm and a rectangle of length 3 cm and breadth 1cm are removed. (as shown in the adjoining figure). Find the area of the remaining sheet. (Take $\pi = \frac{22}{7}$)



Sol: Radius of circular sheet (R) =14 cm

Area of circular sheet $= \pi R^2 = \frac{22}{7} \times 14 \times 14 = 616\ cm^2$

Radius of small circle (r)=3.5cm

Area of small circle $= \pi r^2 = \frac{22}{\cancel{7}} \times \cancel{3.5}^{0.5} \times 3.5$

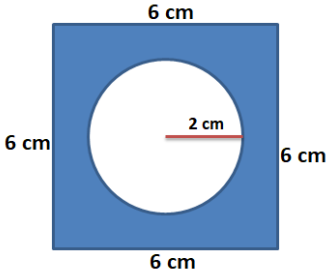
$= 38.5\ cm^2$

Area of rectangle $= 3\ cm \times 1\ cm = 3cm^2$

Area of the remaining sheet = Area of circular sheet − (2 × Area of small circle + Area of rectangle)

$= 616 - (2 \times 38.5 + 3) = 616 - 80 = 536\ cm^2$

11. A circle of radius 2 cm is cut out from a square piece of an aluminium sheet of side 6 cm. What is the area of the left over aluminium sheet? (Take π = 3.14)



Sol: Area of circle = $\pi r^2 = 3.14 \times 2 \times 2 = 12.56\ \text{cm}^2$

Area of square sheet = side × side = $6 \times 6 = 36\text{cm}^2$

The area of the left over aluminium sheet

$= 36 - 12.56 = 23.44\ \text{cm}^2$

12. The circumference of a circle is 31.4 cm. Find the radius and the area of the circle? (Take π = 3.14)

Sol: The circumference of a circle = 31.4 cm

$2\pi r = 31.4$

$2 \times 3.14 \times r = 31.4$

$r = \frac{31.4}{2 \times 3.14} = \frac{10}{2} = 5\ cm$

Radius=5 cm

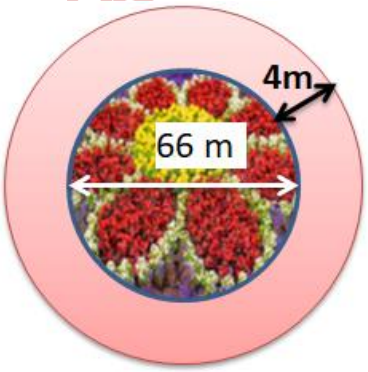
The area of the circle $= \pi r^2 = 3.14 \times 5 \times 5 = 78.5\ cm^2$

13. A circular flower bed is surrounded by a path 4 m wide. The diameter of the flower bed is 66 m. What is the area of this path? (π = 3.14)

Sol: Diameter of flower bed (d)=66 m, radius(r)=33m

Area of the smaller circle $= \pi r^2$

$= 3.14 \times 33 \times 33 = 3419.46\ m^2$



Width of path (w)=4 m

Radius of bigger circle(R) =33+4=37 m

Area of the bigger circle $= \pi R^2$

$= 3.14 \times 37 \times 37 = 4298.66\ m^2$

Area of the path

= Area of bigger circle − Area of smaller circle

$= 4298.66 - 3419.46 = 879.20\ m^2$

(OR)Radius of smaller circle(r)=33 m, Radius of bigger circle(R)=33+4=37m

Area of the path = Area of bigger circle − Area of smaller circle

$= \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R + r)(R - r) = 3.14(37 + 33)(37 - 33)$

$= 3.14 \times 70 \times 4 = 879.20\ m^2$

14. A circular flower garden has an area of 314 m² . A sprinkler at the centre of the garden can cover an area that has a radius of 12 m. Will the sprinkler water the entire garden? (Take π = 3.14)

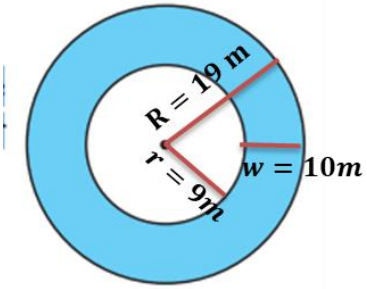
Sol: Area covered by the sprinkler $= \pi r^2 = 3.14 \times 12 \times 12 = 452.16\ m^2$

Area of the circular flower garden=314 m²

Area of circular flower garden is smaller than area covered by the sprinkler.

So, the sprinkler will water the entire garden.

15. Find the circumference of the inner and the outer circles, shown in the adjoining figure? (Take π = 3.14)



Sol: Radius of the outer circle(R)=19 m

Circumference of outer circle $= 2\pi R = 2 \times 3.14 \times 19$
$= 119.32$ m

Radius of the inner circle(r) $= 19 - 10 = 9$ m

Circumference of inner circle $= 2\pi r = 2 \times 3.14 \times 9 = 56.52$ m

16. How many times a wheel of radius 28 cm must rotate to go 352 m? $\left(\text{Take } \pi = \frac{22}{7}\right)$

Sol: Radius of wheel(r)=28 cm

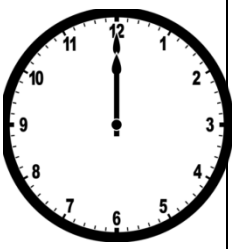
Circumference of the wheel $= 2\pi r$

$= 2 \times \frac{22}{\cancel{7}} \times \cancel{28}^{4} = 176$ cm

Distance=352 m=35200 cm

Number of times wheel should rotate $= \frac{\text{Distance}}{\text{Circumference of the wheel}} = \frac{35200}{176} = 200$

17. The minute hand of a circular clock is 15 cm long. How far does the tip of the minute hand move in 1 hour. (Take π = 3.14)



Sol: Length of minute hand(r)=15 cm

Circumference of circular clock $= 2\pi r = 2 \times 3.14 \times 15 = 94.2\ cm$

The tip of the minute hand move in 1 hour=94.2 cm

CHAPTER 10

VII-MATHEMATICS-NCERT(2023-24)

10. Algebraic Expressions (Notes)

PREPARED BY: BALABHADRA SURESH

https://sureshmathsmaterial.com

1. A variable can take various values. Its value is not fixed.
2. We use letters $x, y, l, m, \ldots$ etc. to denote variables.
3. A constant has a fixed value. Ex: 4, 100, –17, etc
4. **Algebraic expression**: Algebraic expressions are formed from variables and constants.
5. Expressions are made up of terms. Terms are added to make an expression
6. The addition of the terms 4xy and 7 gives the expression 4xy + 7.
7. $x \times x = x^2$
8. $x \times x \times x = x^3$
9. $2 \times y \times y = 2y^2$
10. $4 \times x \times y = 4xy$
11. $2^2 = 2 \times 2 = 4;\ \ 3^2 = 3 \times 3 = 9;\ 4^2 = 4 \times 4 = 16;\ 5^2 = 5 \times 5 = 25$
12. $(-2)^2 = (-2) \times (-2) = 4;\ \ (-3)^2 = (-3) \times (-3) = 9;\ (-4)^2 = (-4) \times (-4) = 16$
13. $2^3 = 2 \times 2 \times 2 = 8;\ \ 3^3 = 3 \times 3 \times 3 = 27;\ 4^3 = 4 \times 4 \times 4 = 64$
14. $(-2)^3 = (-2) \times (-2) \times (-2) = -8;\ \ (-3)^3 = (-3) \times (-3) \times (-3) = -27$

TRY THESE

1. **Describe how the following expressions are obtained**:

(i) $7xy + 5$

Sol: We first obtain xy, multiply it by 7 to get $7xy$ and add 5 to get the expression.

(ii) x^2y

Sol: We first obtain x^2 , and multiply it by y to get x^2y.

(iii) $4x^2 - 5x$

Sol: We first obtain x^2 , and multiply it by 4 to get $4x^2$and subtract $5x$ to get the expression.

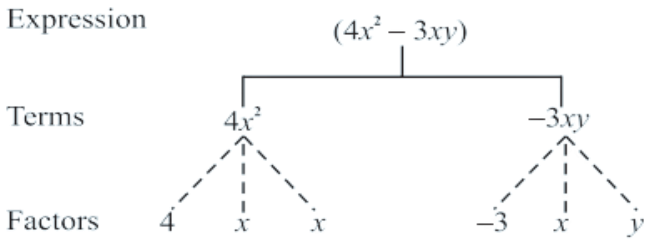
Factors of a term

(i) A term is a product of its factors. The term – $3xy$ is a product of the factors –3, x and y.

(ii) The numerical factor is said to be the numerical coefficient or simply the **coefficient** of the term.

(iii) Factors containing variables are said to be algebraic factors.

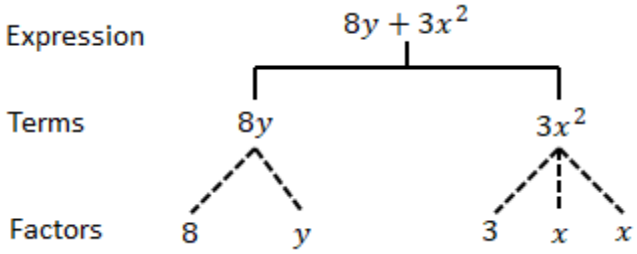
Tree diagram for expression:



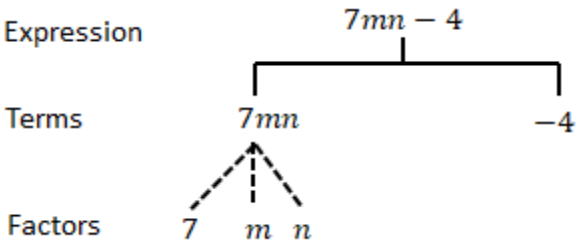
TRY THESE

1. **What are the terms in the following expressions? Show how the terms are formed. Draw a tree diagram for each expression:**

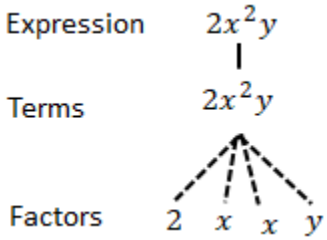
(i) $8y + 3x^2$



(ii) $7mn - 4$



(iii) $2x^2y$



2. **Write three expression each having 4 terms**

Sol: $(i) x^2 + y^2 + xy + 5$

$(ii)\ 2xy + yz - 3xz + 10$

$(iii) - 3x^2 + 4xy + 3x - 7y$

3. Identify the coefficients of the terms of following expressions:

Expression	Terms	Coefficient
$4x - 3y$	$4x$	4
	$-3y$	-3
$a + b + 5$	a	1
	b	1
	5	
$2y + 5$	$2y$	2
	5	
$2xy$	$2xy$	2

Example 1: Identify, in the following expressions, terms which are not constants. Give their numerical coefficients: $xy + 4, 13 - y^2, 13 - y + 5y^2, 4p^2q - 3pq^2 + 5$

Sol:

S. No.	Expression	Term (which is not a Constant)	Numerical Coefficient
(i)	$xy + 4$	xy	1
(ii)	$13 - y^2$	$-y^2$	-1
(iii)	$13 - y + 5y^2$	$-y$ $5y^2$	-1 5
(iv)	$4p^2q - 3pq^2 + 5$	$4p^2q$ $-3pq^2$	4 -3

Example 2: (a) What are the coefficients of x in the following expressions?

$$4x - 3y, 8 - x + y, y^2x - y, 2z - 5xz$$

S. No.	Expression	Term with Factor x	Coefficient of x
(i)	$4x - 3y$	$4x$	4
(ii)	$8 - x + y$	$-x$	-1
(iii)	$y^2x - y$	y^2x	y^2
(iv)	$2z - 5xz$	$-5xz$	$-5z$

(b) What are the coefficients of y in the following expressions?

$$4x - 3y, 8 + yz, yz^2 + 5, my + m$$

S. No.	Expression	Term with factor y	Coefficient of y
(i)	$4x - 3y$	$-3y$	-3
(ii)	$8 + yz$	yz	z
(iii)	$yz^2 + 5$	yz^2	z^2
(iv)	$my + m$	my	m

Like and Unlike Terms

When terms have the same algebraic factors, they are like terms. When terms have different algebraic factors, they are unlike terms.

Try These

Group the like terms together from the following:

$\mathbf{12x, 12, -25x, -25, -25y, 1, x, 12y, y}$

Sol: Group (i): $12x, -25x, \ x$

Group (ii): $-25y, \ 12y, \ y$

Group(iii): 12,1

Types of polynomial:

(1) Monomial: An expression with only one term is called a monomial

Ex: $5xy, \ 7m, \ 2n^2, \ 7x^2y$

(2) Binomial: An expression which contains two unlike terms is called a binomial.

Ex: $7x + 5y, \quad 2m^2 + 5, \ a^2 - b^2$

(3) Trinomial: An expression which contains three terms is called a trinomial

Ex: $x + y + 5, \quad ab + a + b, \ 3x^2 - 5x + 2, \quad m + n + 10$

In general, an expression with one or more terms is called **a polynomial**. Thus a monomial, a binomial and a trinomial are all polynomials.

Example 3: State with reasons, which of the following pairs of terms are of like terms and which are of unlike terms: (i) $\mathbf{7x, 12y}$ (ii) $\mathbf{15x, -21x}$ (iii) $\mathbf{-4ab, 7ba}$ (iv) $\mathbf{3xy, 3x}$

$(v)\ 6xy^2, 9x^2y\ (vi)\ pq^2, -4pq^2 (vii)\ mn^2, 10mn$

S. No.	Pair	Factors	Algebraic factors same or different	Like/ Unlike terms	Remarks
(i)	$7x$ $12y$	$7, x$ $12, y$	Different	Unlike	The variables in the terms are different.
(ii)	$15x$ $-21x$	$15, x$ $-21, x$	Same	Like	
(iii)	$-4ab$ $7\,ba$	$-4, a, b$ $7, a, b$	Same	Like	Remember $ab = ba$
(iv)	$3xy$ $3x$	$3, x, y$ $3, x$	Different	Unlike	The variable y is only in one term.
(v)	$6xy^2$ $9x^2y$	$6, x, y, y$ $9, x, x, y$	Different	Unlike	The variables in the two terms match, but their powers do not match.
(vi)	pq^2 $-4pq^2$	$1, p, q, q$ $-4, p, q, q$	Same	Like	Note, numerical factor 1 is not shown

EXERCISE 10.1

1. Get the algebraic expressions in the following cases using variables, constants and arithmetic operations.

(i) Subtraction of z from y.

Sol: $y - z$

(ii) One-half of the sum of numbers x and y.

Sol: $\frac{1}{2}(x + y)$

(iii) The number z multiplied by itself.

Sol: $z \times z = z^2$

(iv) One-fourth of the product of numbers p and q.

Sol: $\frac{1}{4}pq$

(v) Numbers x and y both squared and added.

Sol: $x^2 + y^2$

(vi) Number 5 added to three times the product of numbers m and n.

Sol: $3mn + 5$

(vii) Product of numbers y and z subtracted from 10.

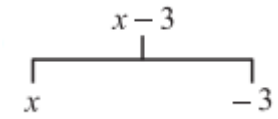
Sol: $10 - yz$

(viii) Sum of numbers a and b subtracted from their product.

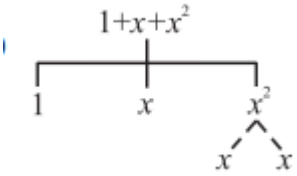
Sol: $ab - (a + b)$

2. **(i) Identify the terms and their factors in the following expressions Show the terms and factors by tree diagrams.**

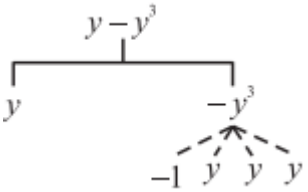
$(a) x - 3$



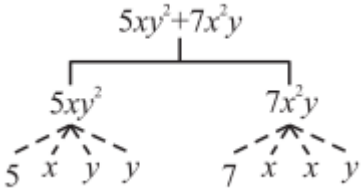
$(b) 1 + x + x^2$



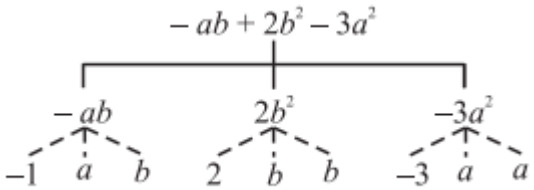
$(c) y - y^3$



$(d) 5xy^2 + 7x^2y$



$(e) - ab + 2b^2 - 3a^2$



(ii) Identify terms and factors in the expressions given below:

$(a) - 4x + 5$ $(b) - 4x + 5y$ $(c) 5y + 3y^2$ $(d) xy + 2x^2y^2$ $(e) pq + q$

$(f) 1.2\,ab - 2.4\,b + 3.6\,a$ $(g) \frac{3}{4}x + \frac{1}{4}$ $(h) 0.1\,p^2 + 0.2\,q^2$

Sol:

	Expression	Terms	Factors
(a)	$-4x + 5$	$-4x$	$-4, x$
		5	5
(b)	$-4x + 5y$	$-4x$	$-4, x$
		$5y$	$5, y$
(c)	$5y + 3y^2$	$5y$	$5, y$
		$3y^2$	$3, y, y$
(d)	$xy + 2x^2y^2$	xy	x, y

		$2x^2y^2$	$2, x, x, y, y$
(e)	$pq + q$	pq	p, q
		q	q
(f)	$1.2ab - 2.4b + 3.6a$	$1.2ab$	$1.2, a, b$
		$-2.4b$	$-2.4, b$
		$3.6a$	$3.6, a$
(g)	$\frac{3}{4}x + \frac{1}{4}$	$\frac{3}{4}x$	$\frac{3}{4}, x$
		$\frac{1}{4}$	
(h)	$0.1p^2 + 0.2q^2$	$0.1p^2$	$0.1, p, p$
		$0.2q^2$	$0.2, q, q$

3. Identify the numerical coefficients of terms (other than constants) in the following expressions:

$(i) 5 - 3t^2$ $(ii) 1 + t + t^2 + t^3$ $(iii) x + 2xy + 3y$ $(iv) 100m + 1000n$

$(v) - p^2q^2 + 7pq$ $(vi) 1.2a + 0.8b$ $(vii) 3.14r^2$ $(viii) 2(l + b)$ $(ix) 0.1y + 0.01y^2$

Sol:

	Expression	Terms	Coefficients
(i)	$5 - 3t^2$	$-3t^2$	-3
(ii)	$1 + t + t^2 + t^3$	t	1
		t^2	1
		t^3	1
(iii)	$x + 2xy + 3y$	x	1
		$2xy$	2
		$3y$	3
(iv)	$100m + 1000n$	$100m$	100
		$1000n$	1000
(v)	$-p^2q^2 + 7pq$	$-p^2q^2$	1
		$7pq$	1
(vi)	$1.2a + 0.8b$	$1.2a$	1.2
		$0.8b$	0.8
(vii)	$3.14r^2$	$3.14r^2$	3.14
(viii)	$2(l + b) = 2l + 2b$	$2l$	2
		$2b$	2
(ix)	$0.1y + 0.01y^2$	$0.1y$	0.1

		$0.01y^2$	0.01

4. (a) Identify terms which contain x and give the coefficient of x.

$(i)\mathbf{y^2\,x\,+\,y}\;(ii)\mathbf{13y^2-8yx}\;(iii)\mathbf{x\,+\,y\,+\,2}\;(iv)\mathbf{5\,+\,z\,+\,zx}\;(v)\mathbf{1\,+\,x\,+\,xy}$

$(vi)\;\mathbf{12xy^2+\,25}\;(vii)\;\mathbf{7x\,+\,xy^2}$

Sol:

	Expression	Terms with x	Coefficient of x
(i)	$y^2x\,+\,y$	y^2x	y^2
(ii)	$13y^2-8yx$	$-8yx$	$-8y$
(iii)	$x\,+\,y\,+\,2$	x	1
(iv)	$5\,+\,z\,+\,zx$	zx	z
(v)	$1\,+\,x\,+\,xy$	$x\ and\ xy$	$1\ and\ y$
(vi)	$12xy^2+\,25$	$12xy^2$	$12y^2$
(vii)	$7\,+\,xy^2$	xy^2	y^2

(b) Identify terms which contain $y^2$and give the coefficient of y^2

$(i)\;\mathbf{8-xy^2}\;(ii)\;\mathbf{5y^2+\,7x}\;(iii)\;\mathbf{2x^2\,y-\,15xy^2\,+\,7y^2}$

	Expression	Terms with y^2	Coefficient of y^2
(i)	$8-xy^2$	$-xy^2$	$-x$
(ii)	$5y^2+\,7x$	$5y^2$	5
(iii)	$2x^2y-\,15xy^2+\,7y^2$	$-15xy^2 and\ 7y^2$	$-15x\ and\ 7$

5. Classify into monomials, binomials and trinomials.

$(i)\mathbf{4y-\,7z}$ → Binomial

$(ii)\mathbf{y^2}$ → Monomial

$(iii)\mathbf{x\,+\,y-\,xy}$ → Trinomial

$(iv)\mathbf{100}$ → Monomial

$(v)\mathbf{ab-\,a-\,b}$ → Trinomial

$(vi)\mathbf{5-\,3t}$ → Binomial

$(vii)\mathbf{4p^2q-\,4pq^2}$ → Trinomial

$(viii)\mathbf{7mn}$ → Monomial

$(ix)\mathbf{z^2-\,3z\,+\,8}$ → Trinomial

$(x)\mathbf{a^2\,+\,b^2}$ → Binomial

$(xi)\mathbf{z^2\,+\,z}$ → Binomial

$(xii)\mathbf{1\,+\,x\,+\,x^2}$ → Trinomial

6. State whether a given pair of terms is of like or unlike terms

$(i)\mathbf{1,100}$ → Like terms

$(ii)-\mathbf{7x},\frac{\mathbf{5}}{\mathbf{2}}\,\mathbf{x}$ → Like terms

$(iii)-\,\mathbf{29x},-\,\mathbf{29y}$ → Unlike terms

$(iv)\mathbf{14xy,42yx}$ → Like terms

$(v)\mathbf{4m^2p,4mp^2}$ → Unlike terms

$(vi)\;\mathbf{12xz,12x^2z^2}$ → Unlike terms

7. Identify like terms in the following:

$(a)-\ xy^2\ ,-\ 4yx^2\ ,8x^2\ \ ,2xy^2\ ,7y,-\ 11x^2,-\ 100x,-\ 11yx,20x^2y,-\ 6x^2,y,2xy,3x$

Sol: $(i)-\ xy^2, 2xy^2\ (ii)\ -\ 4yx^2, 20x^2y\quad (iii)8x^2,-\ 11x^2,-\ 6x^2\ (iv)7y, y$

$(v)-\ 100x, 3x\ \ (vi)\ -\ 11yx, 2xy$

$(b)10pq, 7p, 8q,-\ p^2q^2\ ,-\ 7qp,-\ 100q,-\ 23, 12p^2q^2\ ,-\ 5p^2\ , 41, 2405p, 78qp, 13p^2q, qp^2, 701p^2$

$Sol: (i)10pq, -7qp, 78qp$

$(ii)7p, 2405p\ \ (iii)8q, -100q$

$(iv)-\ p^2q^2, 12p^2q^2\ (v)\ -\ 23{,}41$

$(vi)-\ 5p^2, 701p^2\ (vii)13p^2q, qp^2$

Finding the Value of an Expression

The value of the expression depends on the value of the variable from which the expression is formed.

Example 4:Find the values of the following expressions for $x\ =\ 2$

$(i)\ \ x + 4 = 2 + 4 = 6$

$(ii)\ \ 4x - 3 = (4 \times 2) - 3$

$= 8 - 3 = 5$

$(iii)\ 19 - 5x^2 = 19 - (5 \times 2^2)$

$= 19 - (5 \times 4)$

$= 19 - 20 = -1$

$(iv)\ 100 - 10x^3 = 100 - (10 \times 2^3)$

$= 100 - (10 \times 8)$

$= 100 - 80 = 20$

$2^2 = 2 \times 2 = 4$

$2^3 = 2 \times 2 \times 2 = 8$

Example 5: Find the value of the following expressions when n = – 2.

$(i)\ 5n - 2 = [5 \times (-2)] - 2$

$= -10 - 2 = -12$

$(ii)5n^2 = [5 \times (-2)^2] = (5 \times 4) = 20$

$5n^2 + 5n - 2 = 20 - 12 = 8$

$(iii)\ n^3 = (-2)^3 = (-2) \times (-2) \times (-2) = -8$

$n^3 + 5n^2 + 5n - 2 = -8 + 8 = 0$

Example 6:Find the value of the following expressions for a = 3, b = 2.

$(i)\ \ a\ +\ b = 3 + 2 = 5$

$(ii)\ \ 7a - 4b = 7 \times 3 - 4 \times 2$

$= 21 - 8 = 13$

$(iii)\ \ a^2 + 2ab\ +\ b^2$

$= 3^2 + 2 \times 3 \times 2\ +\ 2^2$

$= 9 + 12 + 4 = 25$

$(iv)\ a^3 - b^3 = 3^3 - 2^3$

$= 3 \times 3 \times 3 - 2 \times 2 \times 2$

$= 9 \times 3 - 4 \times 2$

$= 27 - 8 = 19$

EXERCISE 10.2

1. **If m = 2, find the value of**

$(i)\ \ \boldsymbol{m - 2} = 2 - 2 = 0$

$(ii)\ \ \boldsymbol{3m - 5} = 3 \times 2 - 5$

$= 6 - 5 = 1$

$(iii)\ \ \boldsymbol{9 - 5m} = 9 - 5 \times 2$

$= 9 - 10 = -1$

$(iv)\ \ \boldsymbol{3m^2 - 2m - 7}$

$= 3 \times 2^2 - 2 \times 2 - 7$

$= 3 \times 4 - 4 - 7$

$= 12 - 11 = 1$

$(v)\ \dfrac{\boldsymbol{5m}}{\boldsymbol{2}} - \boldsymbol{4} = \dfrac{5 \times 2}{2} - 4$

$= \dfrac{10}{2} - 4$

$= 5 - 4 = 1$

2. If p = – 2, find the value of:

$(i)\ \ \boldsymbol{4p + 7} = 4 \times (-2) + 7$

$= -8 + 7 = -1$

$(ii)\ \ \boldsymbol{-3p^2} = (-3) \times (-2)^2$

$= (-3) \times 4 = -12$

$\boldsymbol{-3p^2 + 4p + 7} = -12 - 1 = -13$

$(iii)\ \ \boldsymbol{-2p^3} = -2 \times (-2)^3$

$= -2 \times (-8) = +16$

$-3p^2 + 4p + 7 = -13$

$\boldsymbol{-2p^3 - 3p^2 + 4p + 7} = 16 - 13 = 3$

3. **Find the value of the following expressions, when $x = -1$:**

$(i)\ \ \boldsymbol{2x - 7} = 2 \times (-1) - 7$

$= -2 - 7 = -9$

$(ii)\ \ \boldsymbol{-x + 2} = -(-1) + 2$

$= 1 + 2 = 3$

$(iii)\ \boldsymbol{x^2 + 2x + 1} = (-1)^2 + 2(-1) + 1$

$= 1 - 2 + 1 = 2 - 2 = 0$

$(iv)\ \boldsymbol{2x^2 - x - 2} = 2 \times (-1)^2 - (-1) - 2$

$= 2 \times 1 + 1 - 2$

$= 2 + 1 - 2 = 3 - 2 = 1$

$(-2)^2 = (-2) \times (-2) = 4$

4. If a = 2, b = – 2, find the value of:

$(i)\ \ \boldsymbol{a^2 + b^2} = (2)^2 + (-2)^2$

$= 4 + 4 = 8$

$(ii)\ \ \boldsymbol{a^2 + ab + b^2} = (2)^2 + 2 \times (-2) + (-2)^2$

$= 4 - 4 + 4 = 4$

$(i)\ \ \boldsymbol{a^2 - b^2} = (2)^2 - (-2)^2$

$= 4 - 4 = 0$

5. When a = 0, b = – 1, find the value of the given expressions:

$(i)\ \ \boldsymbol{2a + 2b} = 2 \times 0 + 2 \times (-1)$

$= 0 - 2 = -2$

$(-1)^2 = (-1) \times (-1) = 1$

$(ii)\ \ \boldsymbol{2a^2 + b^2 + 1} = 2 \times 0^2 + (-1)^2 + 1$

$= 0 + 1 + 1 = 2$

$(ii)\ \ \boldsymbol{2a^2b + 2ab^2 + ab} = [2 \times 0^2 \times (-1)] + [2 \times 0 \times (-1)^2] + 0 \times (-1)$

$= 0 + 0 + 0 = 0$

$(iii)\ \ \boldsymbol{a^2 + ab + 2} = 0^2 + 0 \times (-1) + 2$

$= 0 - 0 + 2 = 2$

6. Simplify the expressions and find the value if x = 2

We use distributive property in this simplification

$(i)a(b + c) = a \times b + a \times c \ \ and \ \ (ii)a(b - c) = a \times b - a \times c$

$\textbf{(i)}\ \ \boldsymbol{x + 7 + 4(x - 5)} = x + 7 + (4 \times x) - (4 \times 5)$

$= x + 7 + 4x - 20$

$= x + 4x + 7 - 20 = 5x - 13$

$If\ x = 2$

$5x - 13 = (5 \times 2) - 13 = 10 - 13 = -3$

$\textbf{(ii)}\ \ \boldsymbol{3(x + 2) + 5x - 7} = 3 \times x + 3 \times 2 + 5x - 7$

$= 3x + 6 + 5x - 7$

$= 3x + 5x + 6 - 7 = 8x - 1$

$If\ x = 2$

$8x - 1 = (8 \times 2) - 1 = 16 - 1 = 15$

$\textbf{(iii)}\boldsymbol{6x + 5(x - 2)} = 6x + 5 \times x - 5 \times 2$

$= 6x + 5x - 10 = 11x - 10$

$If\ x = 2$

$11x - 10 = (11 \times 2) - 10 = 22 - 10 = 12$

$(iv)\boldsymbol{4(2x - 1) + 3x + 11} = (4 \times 2x) - (4 \times 1) + 3x + 11$

$= 8x - 4 + 3x + 11$

$= 8x + 3x - 4 + 11 = 11x + 7$

$If\ x = 2$

$11x + 7 = (11 \times 2) + 7 = 22 + 7 = 29$

7. Simplify these expressions and find their values if x = 3, a = – 1, b = – 2.

$(i)\ \ \boldsymbol{3x - 5 - x + 9} = 3x - x - 5 + 9$

$= 2x + 4$

If $x = 3$;

$2x + 4 = (2 \times 3) + 4 = 6 + 4 = 10$

(ii) $\mathbf{2 - 8x + 4x + 4} = -8x + 4x + 2 + 4$

$= -4x + 6$

If $x = 3$;

$-4x + 6 = (-4 \times 3) + 6 = -12 + 6 = -6$

(iii) $\mathbf{3a + 5 - 8a + 1} = 3a - 8a + 5 + 1$

$= -5a + 6$

If $a = -1$

$-5a + 6 = (-5 \times -1) + 6 = 5 + 6 = 11$

(iv) $\mathbf{10 - 3b - 4 - 5b} = -3b - 5b + 10 - 4$

$= -8b + 6$

If $b = -2$

$-8b + 6 = (-8 \times -2) + 6 = 16 + 6 = 22$

(v) $\mathbf{2a - 2b - 4 - 5 + a} = 2a + a - 2b - 4 - 5$

$= 3a - 2b - 9$

If $a = -1$ *and* $b = -2$

$3a - 2b - 9 = (3 \times -1) - (2 \times -2) - 9$

$= -3 - (-4) - 9$

$= -3 + 4 - 9 = -11 + 4 = -8$

8. ***(i) If*** $\boldsymbol{z = 10}$, ***find the value of*** $\boldsymbol{z^3 - 3(z - 10)}$.

Sol: $z^3 - 3(z - 10) = z^3 - 3 \times z - 3 \times (-10)$

$= z^3 - 3z + 30$

$If\ z\ =\ 10$

$z^3 - 3z + 30 = 10^3 - 3 \times 10 + 30 = 1000 - 30 + 30 = 1000$

(*ii*)*If* $p\ = -\ 10$, *find the value of* $p^2 - 2p - 100$

Sol: $If\ p\ = -\ 10$

$p^2 - 2p - 100 = (-10)^2 - 2 \times (-10) - 100 = 100 + 20 - 100 = 20$

9. What should be the value of a if the value of $2x^2 + x - a$ equals to 5, when x = 0?

Sol: Given: $2x^2 + x - a = 5$

when $x\ =\ 0$

$2 \times 0^2 + 0 - a = 5$

$0 - a = 5$

$-a = 5$

$a = -5$

10. Simplify the expression and find its value when a = 5 and b = – 3. $2(a^2 + ab) + 3 - ab$

Sol: $2(a^2 + ab) + 3 - ab = (2 \times a^2) + (2 \times ab) + 3 - ab$

$= 2a^2 + 2ab + 3 - ab$

$= 2a^2 + ab + 3$

$When\ a\ =\ 5\ and\ b\ = -\ 3$

$2a^2 + ab + 3 = (2 \times 5^2) + (5 \times -3) + 3$

$= (2 \times 25) - 15 + 3$

$= 50 - 12 = 38$

CHAPTER 11

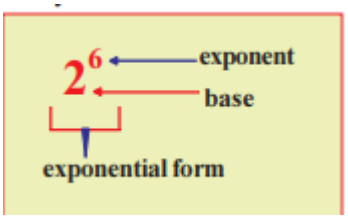
VII-MATHEMATICS-NCERT-2023-24

11. Exponents and Powers (Notes)

REPARED BY: BALABHADRA SURESH

https://sureshmathsmaterial.com

1. These very large numbers are difficult to read, understand and compare. To make these numbers easy to read, understand and compare, we use exponents.
2. 10, 000 = 10 × 10 × 10 × 10 = 10^4 read as 10 raised to the power of 4
3. 100=10×10 =10^2 read as '10 squared'
4. 1000=10×10×10 =10^3 read as '10 cubed'
5. $a \times a = a^2$ (read as 'a squared')
6. $a \times a \times a = a^3$ ($read\ as$ '$a\ cubed$')
7. $a \times a \times a \times a = a^4$ ($read\ as\ a\ raised\ to\ the\ power\ 4$)



$2^2 = 4$	$2^{10} = 1024$	$4^4 = 256$	$7^2 = 49$
$2^3 = 8$	$3^2 = 9$	$4^5 = 1024$	$7^3 = 343$
$2^4 = 16$	$3^3 = 27$	$5^2 = 25$	$8^2 = 64$
$2^5 = 32$	$3^4 = 81$	$5^3 = 125$	$8^3 = 512$
$2^6 = 64$	$3^5 = 243$	$5^4 = 625$	$9^2 = 81$
$2^7 = 128$	$3^6 = 729$	$6^2 = 36$	$9^3 = 729$
$2^8 = 256$	$4^2 = 16$	$6^3 = 216$	$10^2 = 100$
$2^9 = 512$	$4^3 = 64$	$6^4 = 1296$	$10^3 = 1000$

TRY THESE

(i) Express 729 as a power of 3

Sol: $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

(ii) Express 128 as a power of 2

Sol: $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

(iii) Express 343 as a power of 7

Sol: $343 = 7 \times 7 \times 7 = 7^3$

Example 1: Express 256 as a power 2.

Sol: $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$

Example 2:Which one is greater 2^3 or 3^2 ?

Sol: $2^3 = 2 \times 2 \times 2 = 8$ and $3^2 = 3 \times 3 = 9$.

Since 9 > 8

So, 3^2 is greater than 2^3

Example3 : Which one is greater 8^2 *or* 2^8?

Sol: $8^2 = 8 \times 8 = 64$

$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

$\therefore\ 2^8 > 8^2$

Example 4: **Expand $a^3b^2, a^2\,b^3, b^2a^3, b^3a^2$. *Are they all same*?**

Sol: $a^3b^2 = a^3 \times b^2 = a \times a \times a \times b \times b$

$a^2b^3 = a^2 \times b^3 = a \times a \times b \times b \times b$

$b^2a^3 = b^2 \times a^3 = b \times b \times a \times a \times a$

$b^3a^2 = b^3 \times a^2 = b \times b \times b \times a \times a$

$a^3b^2 = b^2a^3$ *and* $a^2b^3 = b^3a^2$

Example 5: **Express the following numbers as a product of powers of prime factors**:

(i) $\mathbf{72} = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

(ii) $\mathbf{432} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$

(iii) $\mathbf{1000} = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^3 \times 5^3$

(iv) $\mathbf{16,000} = 16 \times 1000 = 16 \times 10 \times 10 \times 10$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 = 2^7 \times 5^3$

Example* 6**: ***Work out $\mathbf{(1)^5, (-1)^3, (-1)^4, (-10)^3, (-5)^4}$.

Sol: $(1)^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$

$(-1)^3 = \underline{(-1) \times (-1)} \times (-1) = 1 \times (-1) = -1$

$(-1)^4 = \underline{(-1) \times (-1)} \times \underline{(-1) \times (-1)} = 1 \times 1 = 1$

$(-10)^3 = \underline{(-10) \times (-10)} \times (-10) = 100 \times (-10) = -1000$

$(-5)^4 = \underline{(-5) \times (-5)} \times \underline{(-5) \times (-5)} = 25 \times 25 = 625$

(i) $1^{any\ number} = 1$

(ii) $(-1)^{\text{even number}} = 1$

(iii) $(-1)^{\text{odd number}} = -1$

EXERCISE 11.1

1. Find the value of:

(i) $\mathbf{2^6} = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} = 8 \times 8 = 64$

(ii) $\mathbf{9^3} = \underline{9 \times 9} \times 9 = 81 \times 9 = 729$

(iii) $\mathbf{11^2} = 11 \times 11 = 121$

(iv) $\mathbf{5^4} = 5 \times 5 \times 5 \times 5 = 25 \times 25 = 625$

2. Express the following in exponential form:

(i) $\mathbf{6 \times 6 \times 6 \times 6} = 6^4$

(ii) $\boldsymbol{t \times t} = t^2$

$(iii)\ \boldsymbol{b \times b \times b \times b} = b^4$

$(iv)\ \boldsymbol{5 \times 5 \times 7 \times 7 \times 7} = 5^2 \times 7^3$

$(v)\ \boldsymbol{2 \times 2 \times a \times a} = 2^2 \times a^2$

$(vi)\ \boldsymbol{a \times a \times a \times c \times c \times c \times c \times d} = a^3 \times c^4 \times d$

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

7	343
7	49
7	7
	1

3	729
3	243
3	81
3	27
3	9
3	3
	1

5	3125
5	625
5	125
5	25
5	5
	1

3. Express each of the following numbers using exponential notation:

$(i)\ \boldsymbol{512} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$

$(ii)\ \boldsymbol{343} = 7 \times 7 \times 7 = 7^3$

$(iii)\ \boldsymbol{729} = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

$(iv)\ \boldsymbol{3125} = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$

4. Identify the greater number, wherever possible, in each of the following?

$(i)\ \boldsymbol{4^3\ or\ 3^4}$

Sol: $4^3 = 4 \times 4 \times 4 = 16 \times 4 = 64$

$3^4 = 3 \times 3 \times 3 \times 3 = 9 \times 9 = 81$

$\therefore 3^4\ is\ greater\ than\ 4^3\ (3^4 > 4^3)$

$(ii)\ \boldsymbol{5^3\ or\ 3^5}$

$Sol: 5^3 = 5 \times 5 \times 5 = 25 \times 5 = 125$

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 9 \times 9 \times 3 = 81 \times 3 = 243$

$\therefore 3^5\ is\ greater\ than\ 5^3\ (3^5 > 5^3)$

$(iii)\ \boldsymbol{2^8\ or\ 8^2}$

$Sol:\ 2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 16 \times 16 = 256$

$8^2 = 8 \times 8 = 64$

$\therefore 2^8\ is\ greater\ than\ 8^2 (2^8 > 8^2)$

$(iv)\boldsymbol{100^2\ or\ 2^{100}}$

Sol: $100^2 = 100 \times 100 = 10{,}000$

$2^{100} = 2 \times 2 \times 2 \times 2 \ldots . \times 2(100\ times)$

$= 2 \times 2 \times 2 \times 2 \ldots . \times 2(14\ times) \times 2 \times 2 \times 2 \times 2 \ldots . \times 2(86\ times)$

$= 16384 \times 2 \times 2 \times 2 \times 2 \ldots . \times 2(86\ times)$

$\therefore 2^{100}\ is\ greater\ than 100^2$

$(v)\ \boldsymbol{2^{10}or\ 10^2}$

Sol: $2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 32 \times 32 = 1024$

$10^2 = 10 \times 10 = 100$

$\therefore 2^{10}\ is\ greater\ than\ 10^2$

5. **Express each of the following as product of powers of their prime factors:**

2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

3	405
3	135
3	45
3	15
5	5
	1

2	540
2	270
3	135
3	45
3	15
5	5
	1

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

$(i)\ \boldsymbol{648} = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^3 \times 3^4$

$(ii)\ \boldsymbol{405} = 3 \times 3 \times 3 \times 3 \times 5 = 3^4 \times 5$

$(iii)\ \boldsymbol{540} = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^2 \times 3^3 \times 5$

$(iv)\ \boldsymbol{3,600} = 36 \times 100 = 6 \times 6 \times 10 \times 10$

$= 2 \times 3 \times 2 \times 3 \times 2 \times 5 \times 2 \times 5$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 2^4 \times 3^2 \times 5^2$

6. Simplify:

$(i)\ \boldsymbol{2 \times 10^3} = 2 \times 10 \times 10 \times 10 = 2 \times 1000 = 2000$

$(ii)\ \boldsymbol{7^2 \times 2^2} = 7 \times 7 \times 2 \times 2 = 49 \times 4 = 196$

$(iii)\ \boldsymbol{2^3 \times 5} = 2 \times 2 \times 2 \times 5 = 8 \times 5 = 40$

$(iv)\ \boldsymbol{3 \times 4^4} = 3 \times 4 \times 4 \times 4 \times 4 = 3 \times 256 = 768$

$(v)\ \boldsymbol{0 \times 10^2} = 0 \times 100 = 0$

$(vi)\ \mathbf{5^2 \times 3^3} = 5 \times 5 \times 3 \times 3 \times 3 = 25 \times 27 = 675$

$(vii)\ \mathbf{2^4 \times 3^2} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 16 \times 9 = 144$

$(viii)\ \mathbf{3^2 \times 10^4} = 9 \times 10000 = 90000$

7. Simplify:

$(i)\ \ \mathbf{(-4)^3} = \underline{(-4) \times (-4)} \times (-4) = 16 \times (-4) = -64$

$(ii)\ \mathbf{(-3) \times (-2)^3} = (-3) \times \underline{(-2) \times (-2) \times (-2)} = (-3) \times (-8) = 24$

$(iii)\ \mathbf{(-3)^2 \times (-5)^2} = \underline{(-3) \times (-3)} \times \underline{(-5) \times (-5)} = 9 \times 25 = 225$

$(iv)\ \mathbf{(-2)^3 \times (-10)^3} = (-8) \times (-1000) = 8000$

8. Compare the following numbers:

$(i)\ \mathbf{2.7 \times 10^{12}}\ ;\ \mathbf{1.5 \times 10^8}$

Sol: $2.7 \times 10^{12} = 2.7 \times 10^4 \times 10^8 = 2.7 \times 10000 \times 10^8 = 27000 \times 10^8$

$27000 \times 10^8 > 1.5 \times 10^8$

$\therefore\ 2.7 \times 10^{12} > 1.5 \times 10^8$

$(ii)\mathbf{4 \times 10^{14}}\ ;\ \mathbf{3 \times 10^{17}}$

Sol: $3 \times 10^{17} = 3 \times 10^3 \times 10^{14} = 3000 \times 10^{14}$

$3000 \times 10^{14} > \mathbf{4 \times 10^{14}}$

$\therefore 3 \times 10^{17} > 4 \times 10^{14}$

LAWS OF EXPONENTS

Multiplying Powers with the Same Base

$(i) 2^2 \times 2^3 == 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 2^{2+3}$

$(ii)(-3)^4 \times (-3)^3 = (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) = (-3)^7 = (-3)^{4+3}$

For any non-zero integer$a$, where m *and* n are whole numbers

$a^m \times a^n = a^{m+n}$

TRY THESE

Simplify and write in exponential form

$(i)\ \mathbf{2^5 \times 2^3} = 2^{5+3} = 2^8$

$(ii)\ \mathbf{p^3 \times p^2} = p^{3+2} = p^5$

$(iii)\ \mathbf{4^3 \times 4^2} = 4^{3+2} = 4^5$

$(iv)\ \mathbf{a^3 \times a^2 \times a^7} = a^{3+2+7} = a^{12}$

$(iv)\ \mathbf{5^3 \times 5^7 \times 5^{12}} = 5^{3+7+12} = 5^{22}$

$(v)\ \mathbf{(-4)^{100} \times (-4)^{20}} = (-4)^{100+20} = (-4)^{120}$

Dividing Powers with the Same Base

$(i)\mathbf{3^7 \div 3^4} = \frac{3^7}{3^4} = \frac{3 \times 3 \times 3 \times \cancel{3 \times 3 \times 3 \times 3}}{\cancel{3 \times 3 \times 3 \times 3}} = 3^3 = 3^{7-4}$

$(ii)\mathbf{5^6 \div 5^2} = \frac{5^6}{5^2} = \frac{5 \times 5 \times 5 \times 5 \times \cancel{5 \times 5}}{\cancel{5 \times 5}} = 5^4 = 5^{6-2}$

For any non-zero integer a, where m and n are whole numbers

$a^m \div a^n = a^{m-n}$

TRY THESE

Simplify and write in exponential form:

(i) $\mathbf{2^9 \div 2^3} = 2^{9-3} = 2^6$

(ii) $\mathbf{10^8 \div 10^4} = 10^{8-4} = 10^4$

(iii) $\mathbf{9^{11} \div 9^7} = 9^{11-7} = 9^4$

(iv) $\mathbf{20^{15} \div 20^{13}} = 20^{15-13} = 20^2$

(v) $\mathbf{7^{13} \div 7^{10}} = 7^{13-10} = 7^3$

Taking Power of a Power

(i) $(\mathbf{2^3})^{\mathbf{2}} = 2^3 \times 2^3 = 2^{3+3} = 2^6 = 2^{3\times2}$

(ii) $(\mathbf{3^2})^{\mathbf{4}} = 3^2 \times 3^2 \times 3^2 \times 3^2 = 3^{2+2+2+2} = 3^8 = 3^{2\times4}$

For any non-zero integer 'a', where 'm' and 'n' are whole numbers

$$(a^m)^n = a^{m\times n}$$

TRY THESE

Simplify and write the answer in exponential form:

(i) $(\mathbf{6^2})^{\mathbf{4}} = 6^{2\times4} = 6^8$

(i) $(\mathbf{2^2})^{\mathbf{100}} = 2^{2\times100} = 2^{200}$

(i) $\left(\mathbf{7^{50}}\right)^{\mathbf{2}} = 7^{50\times2} = 7^{100}$

(i) $(\mathbf{5^3})^{\mathbf{7}} = 5^{3\times7} = 5^{21}$

Example 7: Can you tell which one is greater $(5^2) \times 3$ *or* $(5^2)^3$?

Sol: $(5^2) \times 3 = 5 \times 5 \times 3 = 25 \times 3 = 75$

$(5^2)^3 = 5^{2\times3} = 5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 25 \times 25 \times 25 = 625 \times 25 = 15625$

$\therefore (5^2)^3 > (5^2) \times 3$

Multiplying Powers with the Same Exponents

(i) $2^3 \times 3^3 = (2 \times 2 \times 2) \times (3 \times 3 \times 3) = (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= (2 \times 3)^3 = 6^3$

(ii) $a^4 \times b^4 = (a \times a \times a \times a) \times (b \times b \times b \times b)$

$= (a \times b) \times (a \times b) \times (a \times b) \times (a \times b)$

$= (a \times b)^4 = (ab)^4$

For any non-zero integer 'a', where 'm' is any whole number

$a^m \times b^m = (a \times b)^m$

TRY THESE

Put into another form using $a^m \times b^m = (ab)^m$

(i) $\mathbf{4^3 \times 2^3} = (4 \times 2)^3 = 8^3$

(ii) $\mathbf{2^5 \times b^5} = (2 \times b)^5 = (2b)^5$

(iii) $\boldsymbol{a^2 \times t^2} = (a \times t)^2 = (at)^2$

(iv) $\mathbf{5^6 \times (-2)^6} = (5 \times -2)^6 = (-10)^6$

(v) $\mathbf{(-2)^4 \times (-3)^4} = (-2 \times -3)^4 = 6^4$

Example* 8: *Express the following terms in the exponential form:

$(i)(\mathbf{2 \times 3})^5 = 2^5 \times 3^5$

$(ii)(\mathbf{2a})^4 = (2 \times a)^4 = 2^4 \times a^4$

$(iii)(\mathbf{-4m})^3 = (-4 \times m)^3 = (-4)^3 \times m^3$

Dividing Powers with the Same Exponents

$(i)\ \frac{2^4}{3^4} = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^4$

$(ii)\ \frac{a^3}{b^3} = \frac{a \times a \times a}{b \times b \times b} = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} = \left(\frac{a}{b}\right)^3$

For any non-zero integers 'a'and 'b' where 'm' is any whole number

$a^m \div b^m = \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

TRY THESE

Put into another form using $a^m \div b^m = \left(\frac{a}{b}\right)^m$

$(i)\ \mathbf{4^5 \div 3^5} = \left(\frac{4}{3}\right)^5$

$(ii)\ \mathbf{2^5 \div b^5} = \left(\frac{2}{b}\right)^5$

$(iii)\ \mathbf{(-2)^3 \div b^3} = \left(\frac{-2}{b}\right)^3$

$(iv)\ \mathbf{p^4 \div q^4} = \left(\frac{p}{q}\right)^4$

$(v)\ \mathbf{5^6 \div (-2)^6} = \left(\frac{5}{-2}\right)^6$

***Example* 9: *Expand*:**

$(i)\ \left(\frac{\mathbf{3}}{\mathbf{5}}\right)^{\mathbf{4}} = \frac{3^4}{5^4} = \frac{3 \times 3 \times 3 \times 3}{5 \times 5 \times 5 \times 5}$

$(ii)\ \left(\frac{-4}{7}\right)^5 = \frac{(-4)^5}{7^5} = \frac{(-4) \times (-4) \times (-4) \times (-4) \times (-4)}{7 \times 7 \times 7 \times 7 \times 7}$

Numbers with exponent zero

$\frac{3^5}{3^5} = \frac{\cancel{3 \times 3 \times 3 \times 3 \times 3}}{\cancel{3 \times 3 \times 3 \times 3 \times 3}}$

$3^{5-5} = 1$

$3^0 = 1$

For any non − zero integer a, $a^0 = 1$

Example 10: Write exponential form for 8 × 8 × 8 × 8 taking base as 2.

Sol: $8 \times 8 \times 8 \times 8 = 8^4$

$= (2^3)^4 = 2^{3\times4} = 2^{12}$

$(a^m)^n = a^{mn}$

Example 11:Simplify and write the answer in the exponential form.

(i) $\left(\frac{3^7}{3^2}\right) \times 3^5 = 3^{7-2} \times 3^5$

$= 3^5 \times 3^5 = 3^{5+5} = 3^{10}$

$\frac{a^m}{a^n} = a^{m-n}$; $a^m \times a^n = a^{m+n}$

(ii) $2^3 \times 2^2 \times 5^5 = 2^{3+2} \times 5^5$

$a^m \times b^m = (a \times b)^m$

$= 2^5 \times 5^5 = (2 \times 5)^5 = 10^5$

$(iii)(6^2 \times 6^4) \div 6^3 = 6^{2+4} \div 6^3$

$= 6^6 \div 6^3$

$= 6^{6-3} = 6^3$

(iv) $\left[(2^2)^3 \times 3^6\right] \times 5^6 = [2^{2\times3} \times 3^6] \times 5^6$

$= [2^6 \times 3^6] \times 5^6$

$= (2 \times 3)^6 \times 5^6$

$= 6^6 \times 5^6$

$= (6 \times 5)^6 = 30^6$

(v) $8 = 2 \times 2 \times 2 = 2^3$

$8^3 \div 2^3 = (2^3)^3 \div 2^3$

$= 2^9 \div 2^3 = 2^{9-3} = 2^6$

Example 12Simplify:

(i) $\frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27} = \frac{(2^2 \times 3)^4 \times (3^2)^3 \times 2^2}{(2 \times 3)^3 \times (2^3)^2 \times 3^3}$

$= \frac{(2^2)^4 \times 3^4 \times 3^{2\times3} \times 2^2}{2^3 \times 3^3 \times 2^{3\times2} \times 3^3}$

$= \frac{2^8 \times 3^4 \times 3^6 \times 2^2}{2^3 \times 3^3 \times 2^6 \times 3^3}$

$= \frac{2^{8+2} \times 3^{4+6}}{2^{3+6} \times 3^{3+3}} = \frac{2^{10} \times 3^{10}}{2^9 \times 3^6}$

$= 2^{10-9} \times 3^{10-6} = 2^1 \times 3^4$

$= 2 \times 81 = 162$

$(ii)2^3 \times a^3 \times 5a^4 = 2^3 \times 5 \times a^3 \times a^4 = 8 \times 5 \times a^{3+4} = 40a^7$

$(iii)\ \dfrac{\mathbf{2\times 3^4\times 2^5}}{\mathbf{9\times 4^2}}=\dfrac{2\times 3^4\times 2^5}{3^2\times (2^2)^2}$

$=\dfrac{2^{1+5}\times 3^4}{3^2\times 2^4}=\dfrac{2^6\times 3^4}{2^4\times 3^2}$

$=2^{6-4}\times 3^{4-2}=2^2\times 3^2=4\times 9=36$

EXERCISE 11.2

1. Using laws of exponents, simplify and write the answer in exponential form:

1) $a^m\times a^n=a^{m+n}$

2) $a^m\div b^m=\dfrac{a^m}{a^n}=a^{m-n}$

3) $(a^m)^n=a^{mn}$

4) $a^m\times b^m=(a\times b)^m$

5) $\dfrac{a^m}{b^m}=\left(\dfrac{a}{b}\right)^m$

6) $a^0=1\ \ (a\neq 0)$

7) $1^{any\ number}=1$

8) $(-1)^{\text{even number}}=1,$

$(-1)^{\text{odd number}}=-1$

$(i)\ \mathbf{3^2\times 3^4\times 3^8}=3^{2+4+8}=3^{14}$

$(ii)\ \mathbf{6^{15}\div 6^{10}}=6^{15-10}=6^5$

$(iii)\ \mathbf{a^3\times a^2}=a^{3+2}=a^5$

$(iv)\ \mathbf{7^x\times 7^2}=7^{x+2}$

$(v)\ \mathbf{(5^2)^3\div 5^3}=5^{2\times 3}\div 5^3=5^6\div 5^3=5^{6-3}=5^3$

$(vi)\ \mathbf{2^5\times 5^5}=(2\times 5)^5=10^5$

$(vii)\ \mathbf{a^4\times b^4}=(a\times b)^4=(ab)^4$

$(viii)\ \mathbf{(3^4)^3}=3^{4\times 3}=3^{12}$

$(ix)\ \mathbf{(2^{20}\div 2^{15})\times 2^3}=2^{20-15}\times 2^3=2^5\times 2^3=2^{5+3}=2^8$

$(x)\ \mathbf{8^t\div 8^2}=8^{t-2}$

2. Simplify and express each of the following in exponential form:

$(i)\ \dfrac{\mathbf{2^3\times 3^4\times 4}}{\mathbf{3\times 32}}=\dfrac{2^3\times 3^4\times 2^2}{3^1\times 2^5}=\dfrac{2^{3+2}\times 3^4}{3^1\times 2^5}$

$=\dfrac{2^5\times 3^4}{3^1\times 2^5}=3^{4-1}=3^3$

$(ii)[\mathbf{(5^2)^3\times 5^4}]\div \mathbf{5^7}=[5^{2\times 3}\times 5^4]\div 5^7$

$=[5^6\times 5^4]\div 5^7=5^{6+4}\div 5^7$

$=5^{10}\div 5^7=5^{10-7}=5^3$

$(iii)\ \mathbf{25^4 \div 5^3} = (5^2)^4 \div 5^3 = 5^{2\times4} \div 5^3$

$= 5^8 \div 5^3 = 5^{8-3} = 5^5$

$(iv)\ \dfrac{\mathbf{3 \times 7^2 \times 11^8}}{\mathbf{21 \times 11^3}} = \dfrac{3^1 \times 7^2 \times 11^8}{3^1 \times 7^1 \times 11^3}$

$= 7^{2-1} \times 11^{8-3}$

$= 7^1 \times 11^5 = 7 \times 11^5$

$(v)\ \dfrac{\mathbf{3^7}}{\mathbf{3^4 \times 3^3}} = \dfrac{3^7}{3^{4+3}} = \dfrac{3^7}{3^7} = 1$

$(vi)\ \mathbf{2^0 + 3^0 + 4^0} = 1 + 1 + 1 = 3$

$(vii)\ \mathbf{2^0 \times 3^0 \times 4^0} = 1 \times 1 \times 1 = 1$

$(viii)\ \mathbf{(3^0 + 2^0) \times 5^0} = (1 + 1) \times 1 = 2 \times 1 = 2$

$(ix)\ \dfrac{\mathbf{2^8 \times a^5}}{\mathbf{4^3 \times a^3}} = \dfrac{2^8 \times a^5}{(2^2)^3 \times a^3} = \dfrac{2^8 \times a^5}{2^6 \times a^3}$

$= 2^{8-6} \times a^{5-3} = 2^2 \times a^2$

$(x)\ \left(\dfrac{\mathbf{a^5}}{\mathbf{a^3}}\right) \times \mathbf{a^8} = a^{5-3} \times a^8$

$= a^2 \times a^8 = a^{2+8} = a^{10}$

$(xi)\ \dfrac{\mathbf{4^5 \times a^8 b^3}}{\mathbf{4^5 \times a^5 b^2}} = a^{8-5} \times b^{3-2} = a^3 b^1 = a^3 b$

$(xii)\ (\mathbf{2^3 \times 2})^2 = (2^{3+1})^2 = (2^4)^2 = 2^{4\times2} = 2^8$

3. *Say true or false and justify your answer*:

$(i)\ \mathbf{10 \times 10^{11}} = \mathbf{100^{11}} \rightarrow$ False

LHS $= 10^1 \times 10^{11} = 10^{1+11} = 10^{12}$

RHS $= 100^{11}$

LHS $\neq$ RHS

$(ii)\ \mathbf{2^3 > 5^2} \rightarrow False$

LHS $= 2^3 = 8$

RHS $= 5^2 = 25$

LHS $\not>$ RHS

$(iii)\ \mathbf{2^3 \times 3^2 = 6^5} \rightarrow False$

$\text{LHS} = 2^3 \times 3^2 = 8 \times 9 = 72$

$\text{RHS} = 6^5 = \underline{6 \times 6} \times \underline{6 \times 6} \times 6 = 36 \times 36 \times 6 = 1296 \times 6 = 7776$

$\text{LHS} \neq \text{RHS}$

$(iv)\ \ \mathbf{3^0 = 1000^0} \rightarrow \text{True}$

$3^0 = 1\ \ and\ \ 1000^0 = 1$

4. Express each of the following as a product of prime factors only in exponential form:

$(i)\mathbf{108 \times 192} = 2^2 \times 3^3 \times 2^6 \times 3^1 = 2^{2+6} \times 3^{3+1} = 2^8 \times 3^4$

$(ii)\ \mathbf{270} = 27 \times 10 = 3 \times 3 \times 3 \times 2 \times 5 = 2 \times 3^3 \times 5$

$(iii)\mathbf{729 \times 64} = 3^6 \times 2^6$

5. Simplify:

$(i)\ \dfrac{\mathbf{(2^5)^2 \times 7^3}}{\mathbf{8^3 \times 7}} = \dfrac{(2^5)^2 \times 7^3}{(2^3)^3 \times 7}$

$= \dfrac{2^{5\times2} \times 7^3}{2^{3\times3} \times 7^1} = \dfrac{2^{10} \times 7^3}{2^9 \times 7^1}$

$= 2^{10-9} \times 7^{3-1} = 2^1 \times 7^2$

$= 2 \times 49 = 98$

$(ii)\dfrac{\mathbf{25 \times 5^2 \times t^8}}{\mathbf{10^3 \times t^4}} = \dfrac{5^2 \times 5^2 \times t^8}{(2 \times 5)^3 \times t^4}$

$= \dfrac{5^{2+2} \times t^8}{2^3 \times 5^3 \times t^4} = \dfrac{5^4 \times t^8}{2^3 \times 5^3 \times t^4}$

$= \dfrac{5^{4-3} \times t^{8-4}}{2^3} = \dfrac{5^1 \times t^4}{2^3} = \dfrac{5t^4}{8}$

$(iii)\dfrac{\mathbf{3^5 \times 10^5 \times 25}}{\mathbf{5^7 \times 6^5}} = \dfrac{3^5 \times (2 \times 5)^5 \times 5^2}{5^7 \times (2 \times 3)^5}$

$= \dfrac{3^5 \times 2^5 \times 5^5 \times 5^2}{5^7 \times 2^5 \times 3^5}$

$= \dfrac{3^5 \times 2^5 \times 5^7}{5^7 \times 2^5 \times 3^5} = 1$

DECIMAL NUMBER SYSTEM

(i) $47561 = 4 \times 10000 + 7 \times 1000 + 5 \times 100 + 6 \times 10 + 1$

$= 4 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 1 \times 10^0$

(ii) $104278 = 1 \times 100{,}000 + 0 \times 10{,}000 + 4 \times 1000 + 2 \times 100 + 7 \times 10 + 8 \times 1$

$= 1 \times 10^5 + 0 \times 10^4 + 4 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 8 \times 10^0$

TRY THESE

Expand by expressing powers of 10 in the exponential form:

(i) 172=1×100+7×10+2×1

$= 1 \times 10^2 + 7 \times 10^1 + 2 \times 10^0$

(ii) 5,643=5×1000+6×100+4×10+3×1

$= 5 \times 10^3 + 6 \times 10^2 + 4 \times 10^1 + 3 \times 10^0$

(iii) 56,439=5 × 10,000 + 6 × 1000 + 4 × 100 + 3 × 10 + 9 × 1

$= 5 \times 10^4 + 6 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$

(iv) 1,76,428=1×1,00,000+7 × 100,000 + 6 × 10,000 + 4 × 1000 + 2 × 100 + 8 × 10 + 8 × 1

$= 1 \times 10^5 + 7 \times 10^4 + 6 \times 10^3 + 4 \times 10^2 + 2 \times 10^1 + 8 \times 10^0$

EXPRESSING LARGE NUMBERS IN THE STANDARD FORM

Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its standard form.

(i) $59 = 5.9 \times 10 = 5.9 \times 10^1$

(ii) $590 = 5.9 \times 100 = 5.9 \times 10^2$

(iii) $5985 = 5.985 \times 1000 = 5.985 \times 10^3$

(iv) The distance between Sun and Saturn is 1,433,500,000,000 m=1.4335×10^{12} m

(v) The distance between Saturn and Uranus is 1,439,000,000,000 m=1.439×10^{12} m

(vi) distance between Sun and Earth is 149, 600,000,000 m = 1.496×10^{11}m

EXAMPLE 13Express the following numbers in the standard form:

(i) 5985.3 = 5.9853 × 1000 = 5.9853×10^3

(ii) 65,950 = 6.595 × 10,000 = 6.595×10^4

(iii) 3,430,000 = 3.43 × 1,000,000 = 3.43×10^6

(iv) 70,040,000,000 = 7.004 × 10,000,000,000 = 7.004×10^{10}

EXERCISE 11.3

1. **Write the following numbers in the expanded forms**:

(i) $\mathbf{279404} = 2 \times 10^5 + 7 \times 10^4 + 9 \times 10^3 + 4 \times 10^2 + 0 \times 10^1 + 4 \times 10^0$

(ii) $\mathbf{3006194} = 3 \times 10^6 + 0 \times 10^5 + 0 \times 10^4 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$

(iii) $\mathbf{2806196} = 2 \times 10^6 + 8 \times 10^5 + 0 \times 10^4 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 6 \times 10^0$

(iv) $\mathbf{120719} == 1 \times 10^5 + 2 \times 10^4 + 0 \times 10^3 + 7 \times 10^2 + 1 \times 10^1 + 9 \times 10^0$

(v) $\mathbf{20068} = 2 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 6 \times 10^1 + 8 \times 10^0$

2. **Find the number from each of the following expanded forms:**

(a) $8\times10^4 + 6\times10^3 + 0\times10^2 + 4\times10^1 + 5\times10^0 = 86045$

(b) $4\times10^5 + 5\times10^3 + 3\times10^2 + 2\times10^0 = 405302$

(c) $3\times10^4 + 7\times10^2 + 5\times10^0 = 30705$

(d) $9\times10^5 + 2\times10^2 + 3\times10^1 = 900230$

3. Express the following numbers in standard form:

(i) $\mathbf{5,00,00,000} = 5\times10^7$

(ii) $\mathbf{70,00,000} = 7\times10^6$

(iii) $\mathbf{3,18,65,00,000} = 3.1865\times10^9$

(iv) $\mathbf{3,90,878} = 3.90878\times10^5$

(v) $\mathbf{39087.8} = 3.90878\times10^4$

(vi) $\mathbf{3908.78} = 3.90878\times10^3$

4. Express the number appearing in the following statements in standard form.

(a) The distance between Earth and Moon is 384,000,000 m

Sol: $3.84\times10^8\ m$

(b) Speed of light in vacuum is 300,000,000 m/s.

Sol: $3\times10^8 m/s$

(c) Diameter of the Earth is 1,27,56,000 m

Sol: $1.2756\times10^7\ m$

(d) Diameter of the Sun is 1,400,000,000 m

Sol: $1.4\times10^9 m$

(e) In a galaxy there are on an average 100,000,000,000 stars.

Sol: 1×10^{11}

(f) The universe is estimated to be about 12,000,000,000 years old.

Sol: $1.2\times10^{10}\ years$

(g) The distance of the Sun from the centre of the Milky Way Galaxy is estimated to be 300,000,000,000,000,000,000 m.

Sol: $3\times10^{20} m$

(h) 60,230,000,000,000,000,000,000 molecules are contained in a drop of water weighing 1.8 gm.

Sol: $6.023\times10^{22}\ molecules$

(i) The earth has 1,353,000,000 cubic km of sea water.

Sol: $1.353\times10^9 cubik\ km\ or\ km^3$

(j) The population of India was about 1,027,000,000 in March, 2001.

Sol: 1.027×10^9

CHAPTER 12

VII-MATHEMATICS-NCERT-2023-24

12. Symmetry(Notes)

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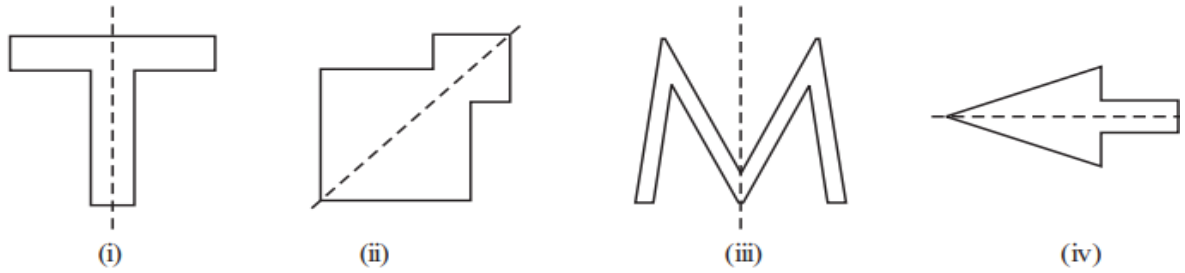
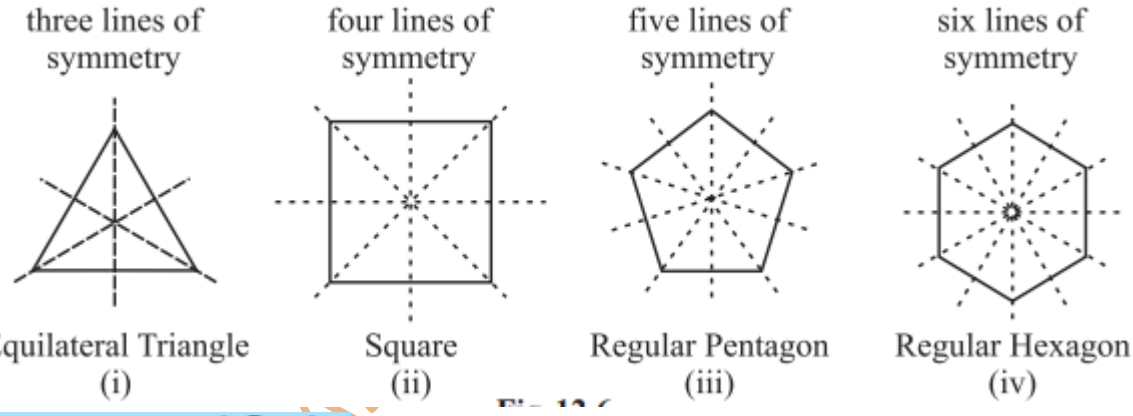
https://sureshmathsmaterial.com

1. If a line divides the given figure into two coincidental parts, then the figure is said to be 'symmetrical' and the line is called the 'axis of symmetry' or 'line of symmetry'.
2. A figure can have one or more than one lines of symmetry or axes of symmetry.



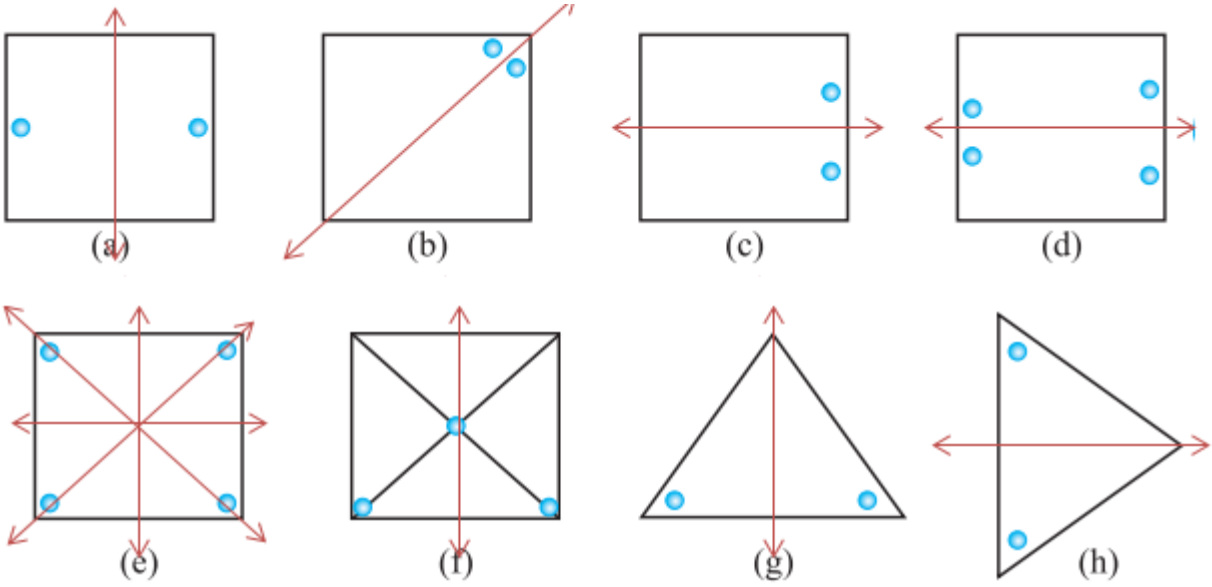
LINES OF SYMMETRY FOR REGULAR POLYGONS

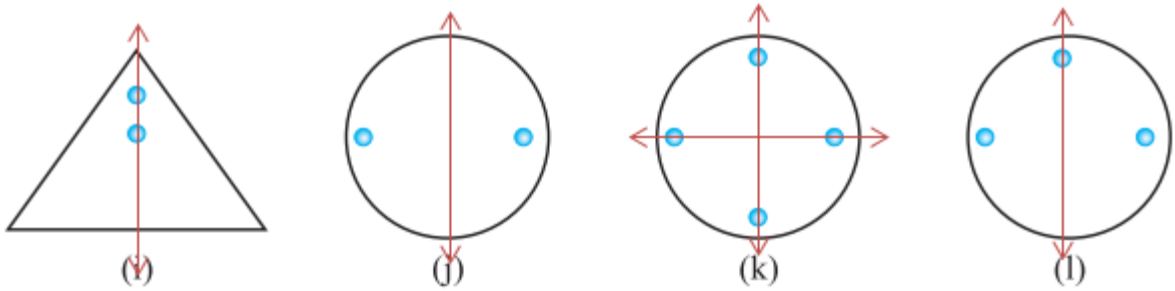
3. A polygon is said to be regular if all its sides are of equal length and all its angles are of equal measure,
4. An equilateral triangle is a regular polygon of three sides.
5. A square is a regular polygon of four sides.
6. Regular polygons have multiple (i.e., more than one) lines of symmetry.



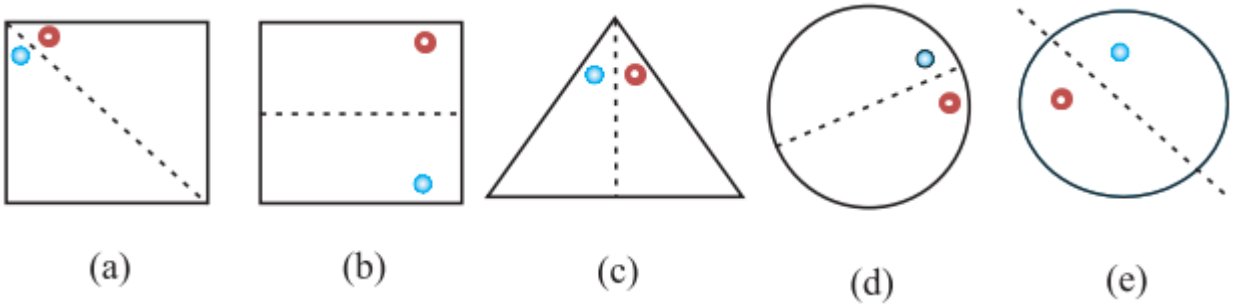
EXERCISE 12.1

1. **Copy the figures with punched holes and find the axes of symmetry for the following:**

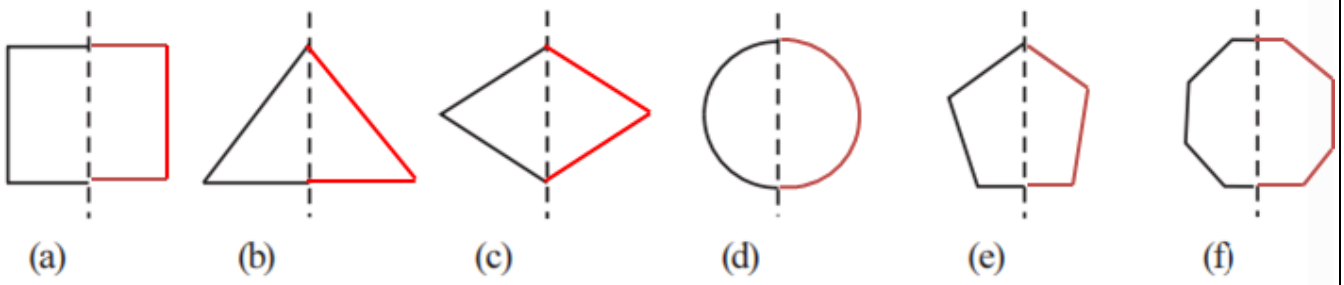




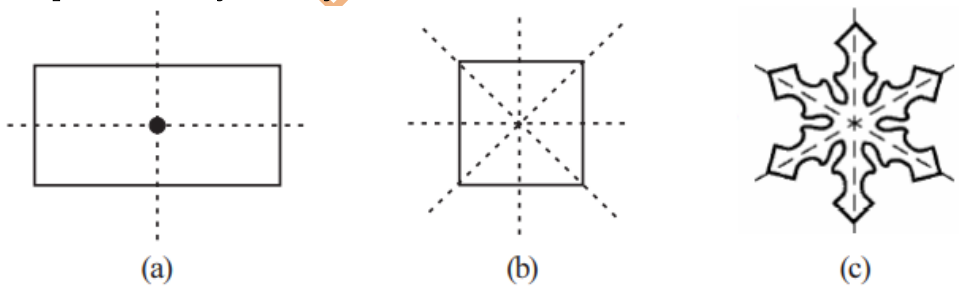
2. Given the line(s) of symmetry, find the other hole(s):



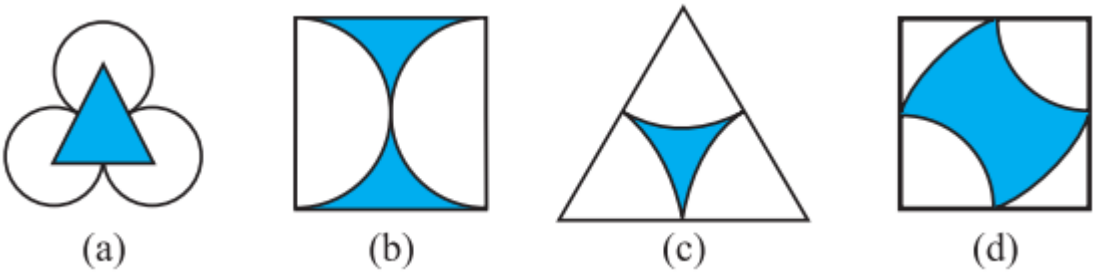
3. In the following figures, the mirror line (i.e., the line of symmetry) is given as a dotted line. Complete each figure performing reflection in the dotted (mirror) line. (You might perhaps place a mirror along the dotted line and look into the mirror for the image). Are you able to recall the name of the figure you complete?

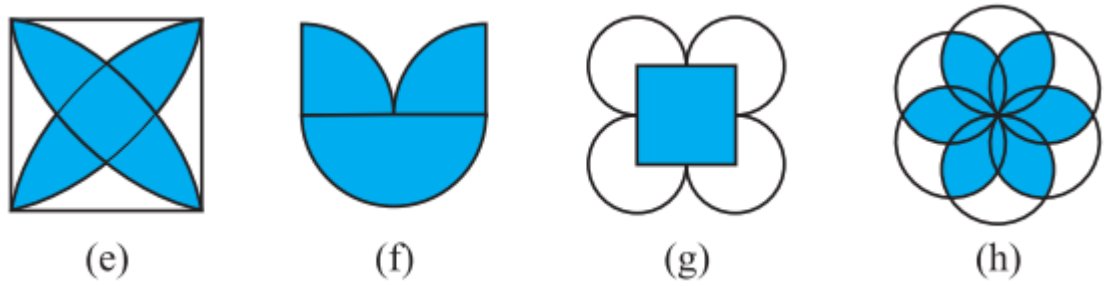


4. The following figures have more than one line of symmetry. Such figures are said to have multiple lines of symmetry.

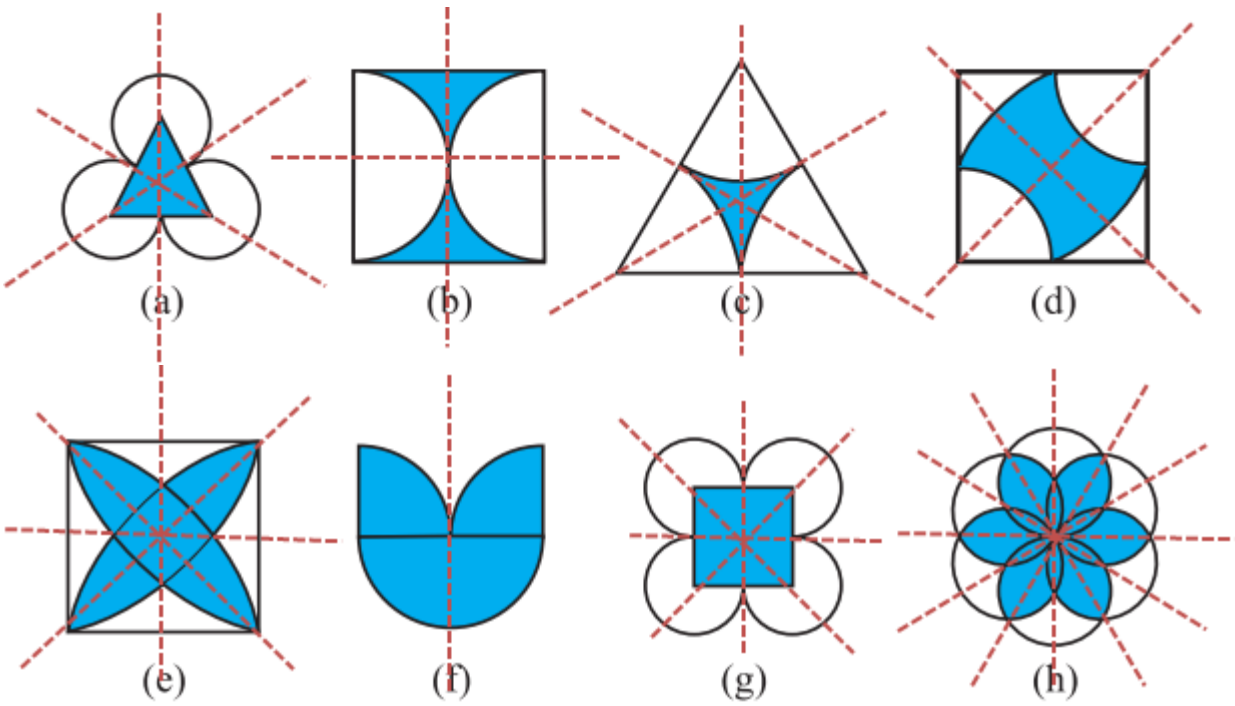


Identify multiple lines of symmetry, if any, in each of the following figures:

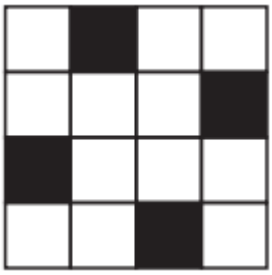




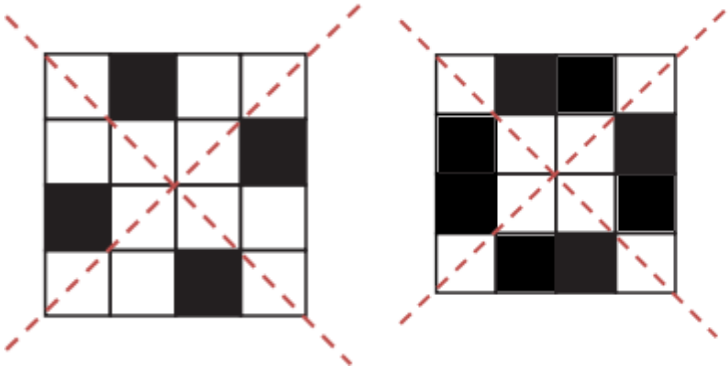
Sol:



5. Copy the figure given here.
Take any one diagonal as a line of symmetry and shade a few more squares to make the figure symmetric about a diagonal. Is there more than one way to do that? Will the figure be symmetric about both the diagonals?

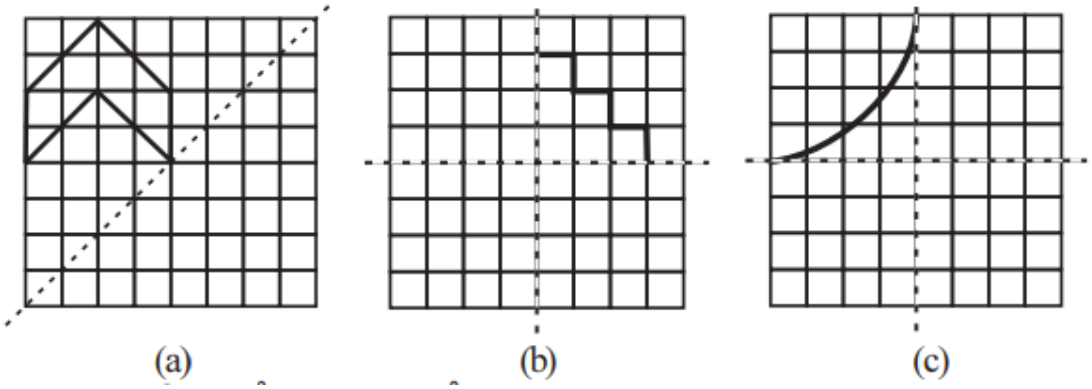


Sol: Yes, the figure is symmetric about the diagonals.

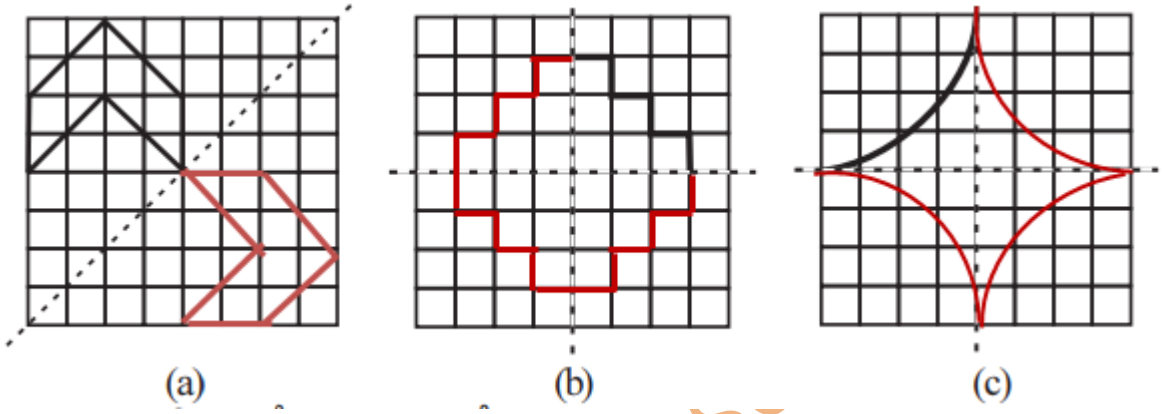


There is more than one way so as to make the figure symmetric about a diagonal as we can choose any of its 2 diagonals.

6. Copy the diagram and complete each shape to be symmetric about the mirror line(s):



Sol:



7. State the number of lines of symmetry for the following figures:

(a) An equilateral triangle -3
(b) An isosceles triangle -1
(c) A scalene triangle -0
(d) A square -4
(e) A rectangle-2
(f) A rhombus -2
(g) A parallelogram -0
(h) A quadrilateral -0
(i) A regular hexagon -6
(j) A circle-Infinitely many

8. What letters of the English alphabet have reflectional symmetry (i.e., symmetry related to mirror reflection) about?

(a) a vertical mirror
Sol: A,H,I,M,O,T,U,V,W,X,Y
(b) a horizontal mirror
Sol: B,C,D,E,H,I,K,O,X
(c) both horizontal and vertical mirrors.
Sol: H,I,O,X

9. **Give three examples of shapes with no line of symmetry.**

Sol: (1) A Scalene triangle (2) A Quadrilateral (3) A Parallelogram.

10. What other name can you give to the line of symmetry of (a) an isosceles triangle? (b) a circle?

Sol: (a) Median or altitude.

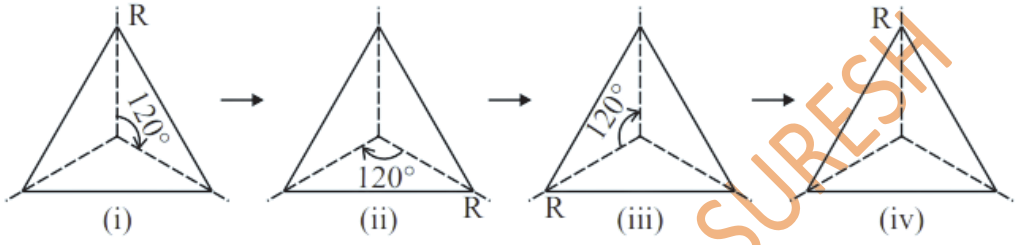
(b) Diameter.

12.3 Rotational Symmetry

(i) If, after a rotation, an object looks exactly the same, we say that it has a rotational symmetry.
(ii) The rotation turns an object about a fixed point. This fixed point is the centre of rotation.
(iii) The angle by which the object rotates is called the angle of rotation.
(iv) In a complete turn (of 360^0), the number of times an object looks exactly the same is called the order of rotational symmetry.
(v) A half-turn means rotation by 180°; a quarter-turn is rotation by 90°.
(vi) The order of symmetry of a square is 4 .
(vii) The order of symmetry of an equilateral triangle is 3.

Try These

1. (a) Can you now tell the order of the rotational symmetry for an equilateral triangle?



Sol: The order of the rotational symmetry for an equilateral triangle is 3

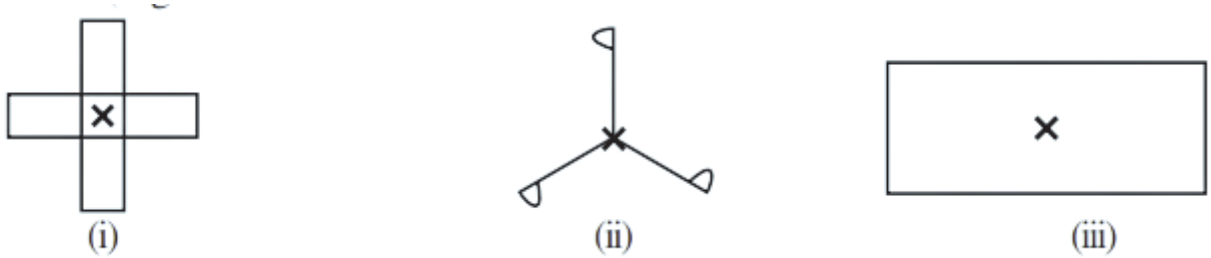
(b) How many positions are there at which the triangle looks exactly the same, when rotated about its centre by 120°?
Sol: 3

2. Which of the following shapes (Fig 12.15) have rotational symmetry about the marked point.

Sol: (i) 2 (ii) 4 (iii) 1 (iv) 5

Try These

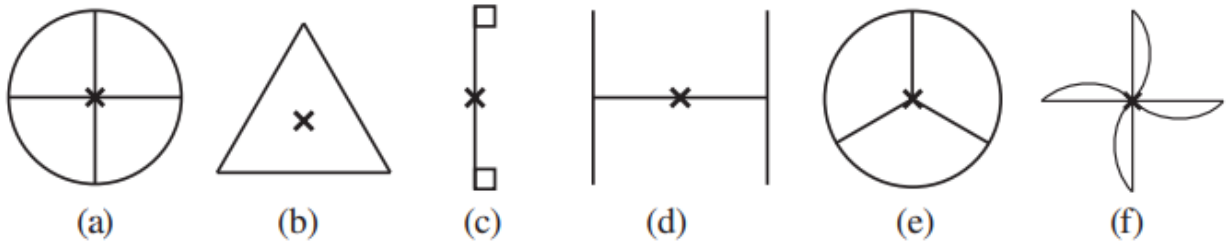
Give the order of the rotational symmetry of the given figures about the point marked (Fig 12.17).



Sol: (i) 4 (ii) 3 (iii) 2

EXERCISE 12.2

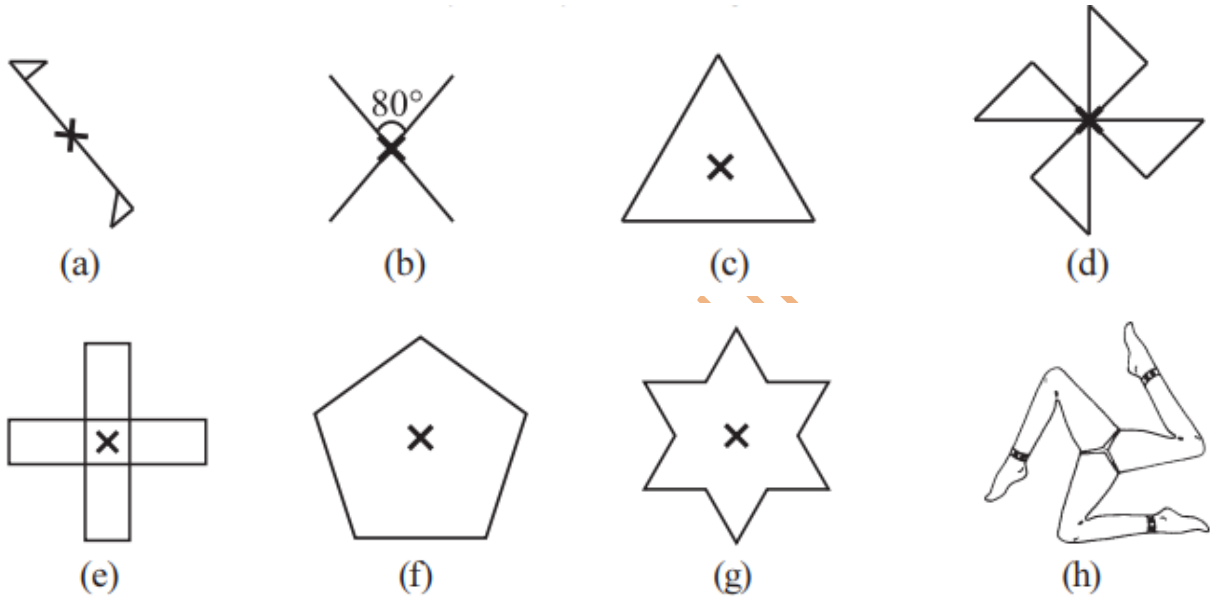
1. Which of the following figures have rotational symmetry of order more than 1:



(a) (b) (c) (d) (e) (f)

Sol: Order of rotational symmetry for (a) 4, (b) 3, (c) 1, (d) 2,(e)3, (f)4.

2. Give the order of rotational symmetry for each figure:



(a) (b) (c) (d)

(e) (f) (g) (h)

Sol:

Figure	order of rotational symmetry
(a)	1
(b)	2
(c)	3
(d)	4
(e)	4
(f)	5
(g)	6
(h)	3

12.4 LINE SYMMETRY AND ROTATIONAL SYMMETRY

Some shapes have only line symmetry (Ex: **E**), some have only rotational symmetry (Ex: **S**) and some have both line symmetry and rotational symmetry (Ex: **H**)

DO THIS

Fill the blanks in the following table:

Alphabet Letters	Line Symmetry	Number of Lines of Symmetry	Rotational Symmetry	Order of Rotational Symmetry
Z	No	0	Yes	2
S	No	0	Yes	2
H	Yes	2	Yes	2
O	Yes	Infinite	Yes	Infinite
E	Yes	1	No	1
N	No	0	Yes	2
C	Yes	1	No	1

EXERCISE 12.3

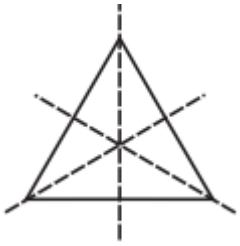
1. **Name any two figures that have both line symmetry and rotational symmetry.**

Sol: Square, Circle, Equilateral triangle,...

2. **Draw, wherever possible, a rough sketch of**

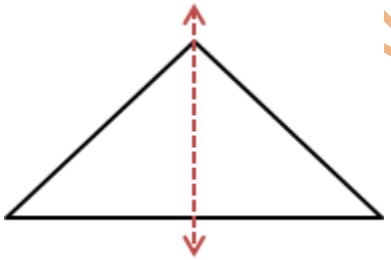
(i) **a triangle with both line and rotational symmetries of order more than 1.**

Sol: An equilateral triangle. (Order of line symmetry is 3 and order of rotational symmetry is 3)



(ii) a triangle with only line symmetry and no rotational symmetry of order more than 1.

Sol: Isosceles triangle (Order of line symmetry is 1 and order of rotational symmetry is 1)

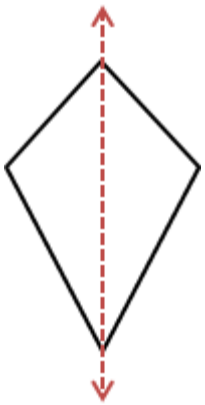


(iii) a quadrilateral with a rotational symmetry of order more than 1 but not a line symmetry.

Sol: Parallelogram (order of rotational symmetry is 2 but no line symmetry)

(iv) a quadrilateral with line symmetry but not a rotational symmetry of order more than 1.

Sol: A kite



3. If a figure has two or more lines of symmetry, should it have rotational symmetry of order more than 1?

Sol: Yes.

4. Fill in the blanks:

Shape	Centre of Rotation	Order of Rotation	Angle of Rotation
Square	Intersecting point of diagonals	4	90^0
Rectangle	Intersecting point of diagonals	2	180^0
Rhombus	Intersecting point of diagonals	2	180^0
Equilateral Triangle	Intersecting point of medians	3	120^0
Regular Hexagon	Intersecting point of diagonals	6	60^0
Circle	Centre of the circle	Infinite	At every point
Semi-circle	Centre of the circle	1	360^0

5. Name the quadrilaterals which have both line and rotational symmetry of order more than 1.

Sol: Square.

6. After rotating by 60° about a centre, a figure looks exactly the same as its original position. At what other angles will this happen for the figure?

Sol: $120^0, 180^0, 240^0, 300^0, 360^0$ (Multiples of 60^0)

7. Can we have a rotational symmetry of order more than 1 whose angle of rotation is (i) 45°? (ii) 17°?

Sol: (i)Yes (45^0 is a factor of 360^0)

(ii) No (17^0 is not a factor of 360^0)

CHAPTER 13

VII-MATHEMATICS-NCERT-2023-24

13. Visualising Solid Shapes (Notes)

PREPARED BY: BALABHADRA SURESH

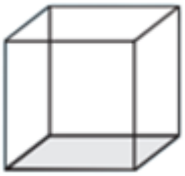
https://sureshmathsmaterial.com

1. You will classify figures you have seen in terms of what is known as dimension.
2. Figures drawn on paper which have only length and breadth are called two dimensional(2-D) (i.e., plane) figures
3. The circle, the square, the rectangle, the quadrilateral and the triangle are examples of plane figures;
4. Some objects have some length, breadth and height or depth. They have three dimensions. They are called three dimensional (3-D) shapes.
5. The cube, the cuboid, the sphere, the cylinder, the cone and the pyramid are examples of solid shapes.

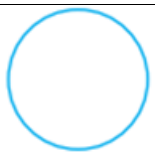
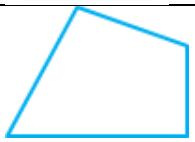
https://sureshmathsmaterial.com/

TRY THESE

Match the shape with the names.

Shape	Name	Shape	Name
	Cylinder		Cube
	Sphere		Cone
	Cuboid		Pyramid

Match the 2 dimensional figures with the names

2 dimensional figure	Name	2 dimensional figure	Name
	Rectangle		Square
	Circle		Quadrilateral
	Triangle		

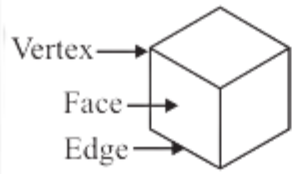
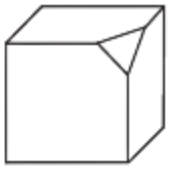
FACES, EDGES AND VERTICES

Faces: Faces are flat, two-dimensional surfaces that make up the boundary of a solid.
Edges: Edges are the one-dimensional lines that form the boundaries of the faces.
Vertices: Vertices (singular: vertex) are the corner points of a solid where edges meet

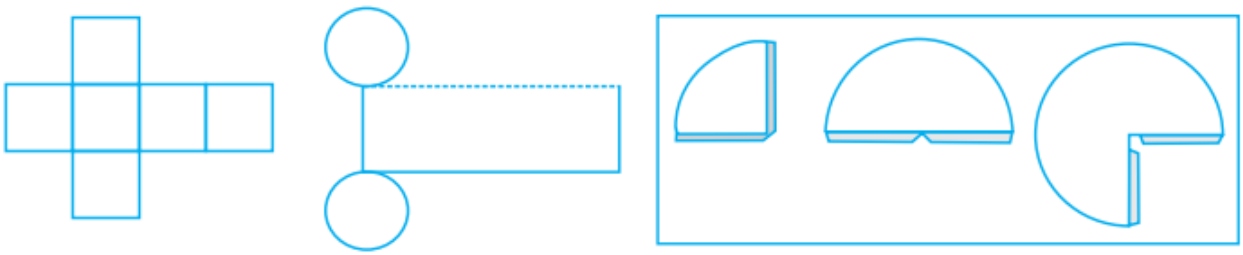
Do this

Complete the following table:

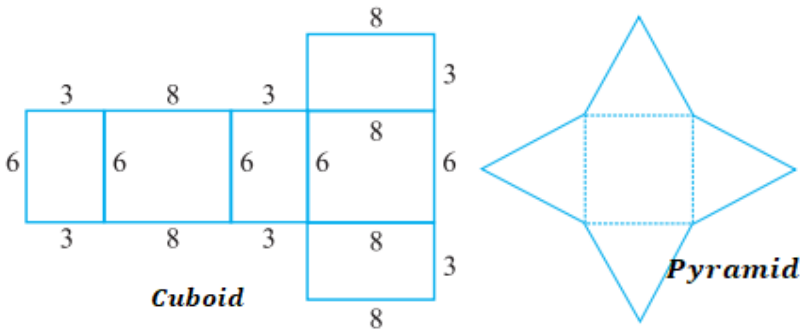
Table 13.1

				
Faces (F)	6	4	9	7
Edges (E)	12	6	16	15
Vertices (V)	8	4	9	10

NETS FOR BUILDING 3-D SHAPES



Cube (i) | Cylinder (ii) | Cone (iii)



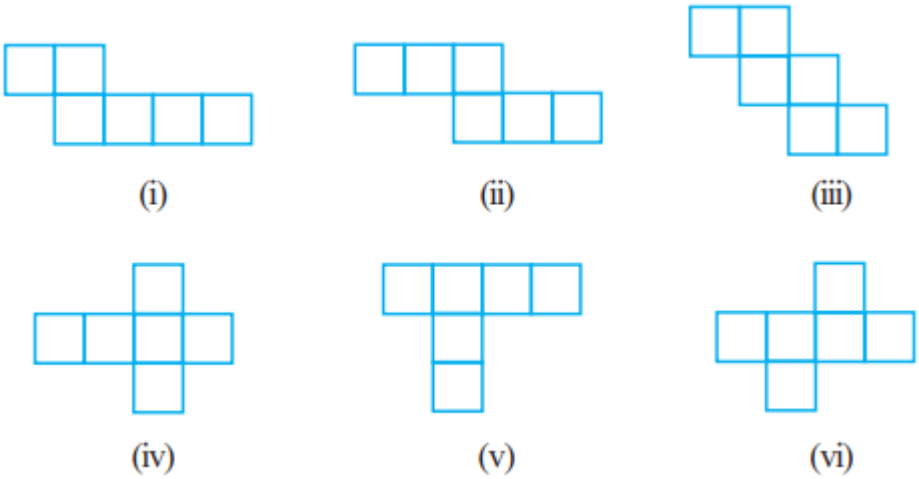
TRY THESE

Here you find four nets (Fig 13.10). There are two correct nets among them to make a tetrahedron. See if you can work out which nets will make a tetrahedron.

Sol: First and third net will make a tetrahedron.

EXERCISE 13.1

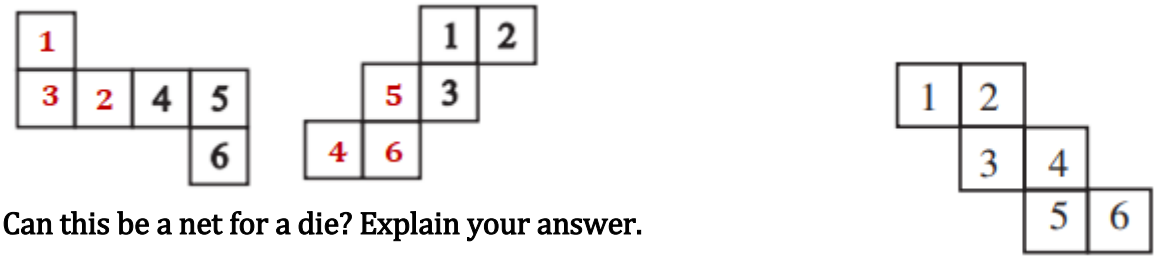
1. **Identify the nets which can be used to make cubes (cut out copies of the nets and try it):**



Sol: Nets in (ii), (iii), (iv) and (vi) form cubes.

2. **Dice are cubes with dots on each face. Opposite faces of a die always have a total of seven dots on them. Here are two nets to make dice (cubes); the numbers inserted in each square indicate the number of dots in that box. Insert suitable numbers in the blanks, remembering that the number on the opposite faces should total to 7**

Sol:



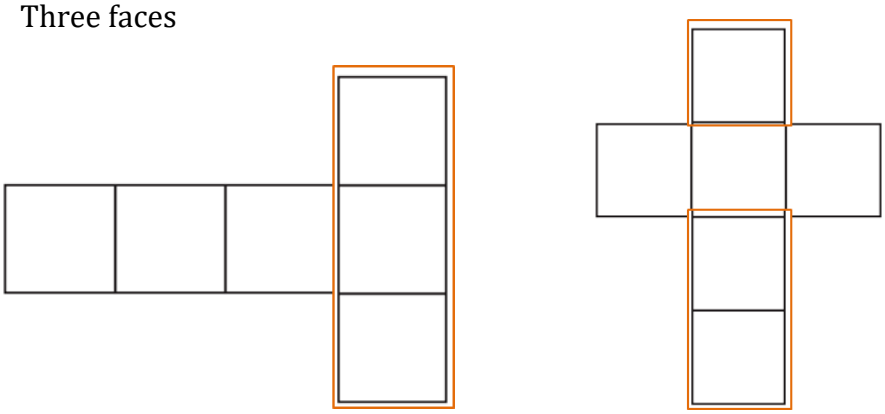
3. **Can this be a net for a die? Explain your answer.**

Sol: No, We know that opposite faces of a die always have a total of seven dots on them.

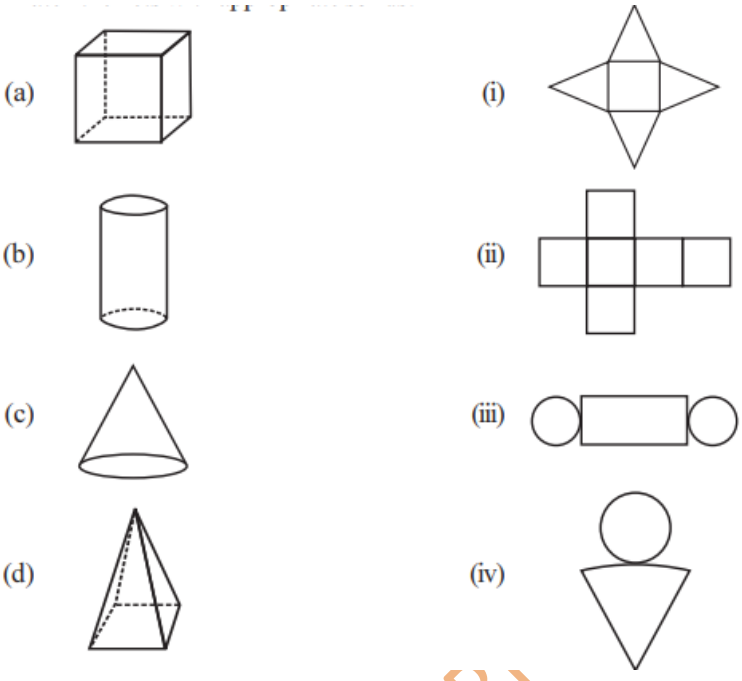
If we will fold this net opposite to 4 will be 1 which does not make total of 7.

4. **Here is an incomplete net for making a cube. Complete it in at least two different ways. Remember that a cube has six faces. How many are there in the net here? (Give two separate diagrams. If you like, you may use a squared sheet for easy manipulation.)**

Sol: Three faces



5. **Match the nets with appropriate solids:**



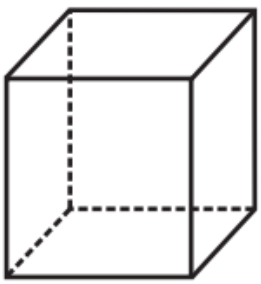
Sol: (a) →(ii) (b)→ (iii) (c)→ (iv) (d)→ (i)

13.4 DRAWING SOLIDS ON A FLAT SURFACE

13.4.1 Oblique Sketches

An oblique sketch does not have proportional lengths. Still it conveys all important aspects of the appearance of the solid.

If we are able to recognise a sketch of a solid is called an oblique sketch.



The sketch of a cube.

In the oblique sketch above you note the following.

(i) The sizes of the front faces and its opposite are same; and

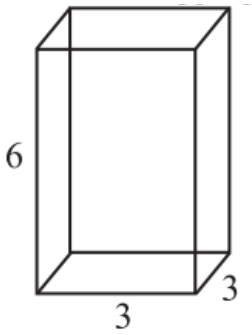
(ii) The edges, which are all equal in a cube, appear so in the sketch, though the actual measures of edges are not taken so.

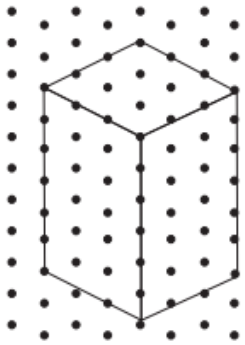
13.4.2 Isometric Sketches

An isometric sketch is drawn on an isometric dot paper. In an isometric sketch of the solid the measurements kept proportional.

In the isometric dot sheet divides the paper into small equilateral triangles made up of dots or lines.

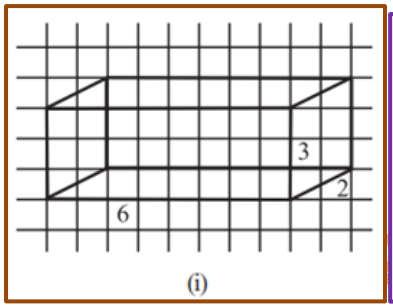
EXAMPLE 1: Here is an oblique sketch of a cuboid [Fig 13.14(i)]. Draw an isometric sketch that matches this drawing. SOLUTION Here is the solution [Fig 13.14(ii)]. Note how the measurements are taken care of.



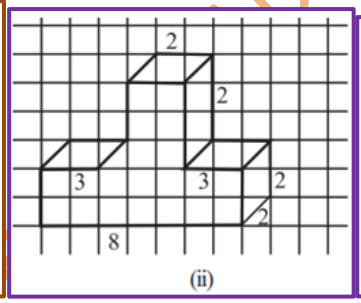


EXERCISE 13.2

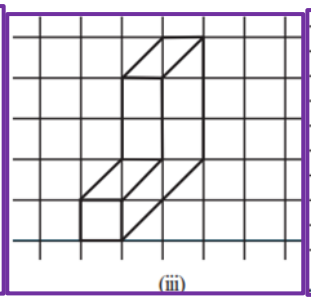
1. Use isometric dot paper and make an isometric sketch for each one of the given shapes:

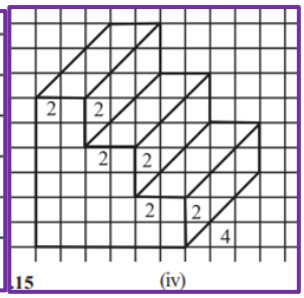


(i)

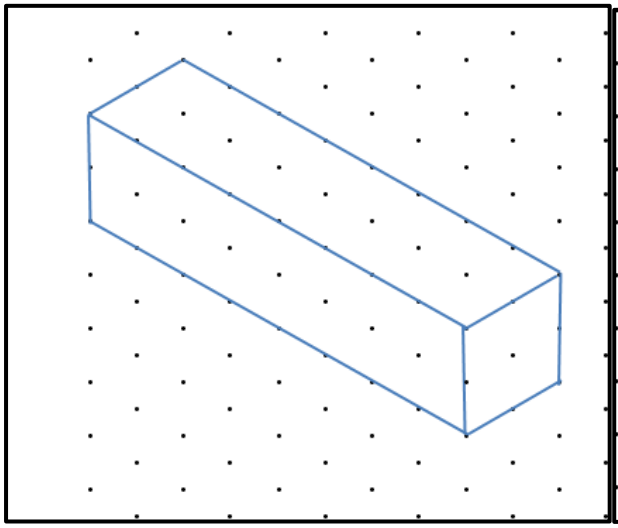


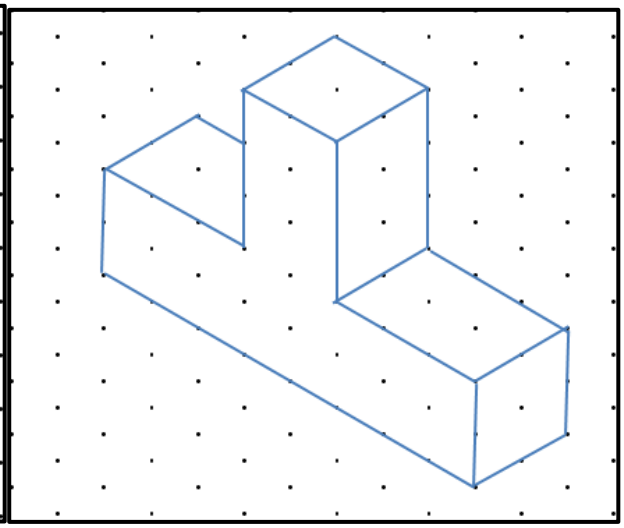
(ii)

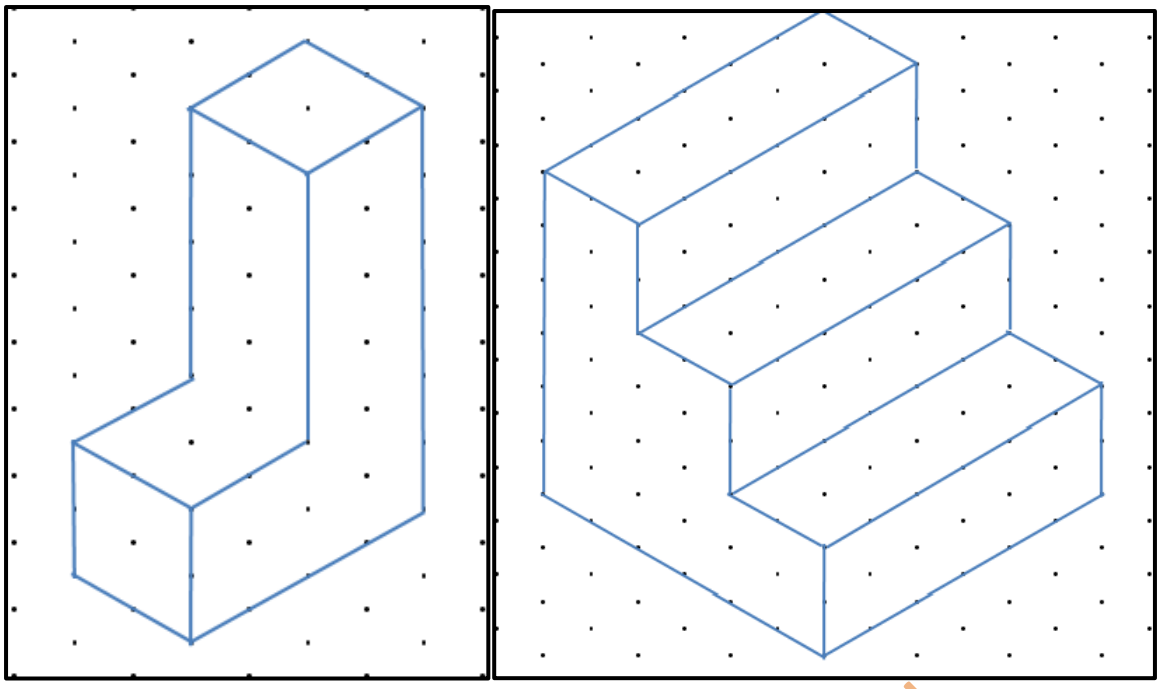

(iii)



(iv)

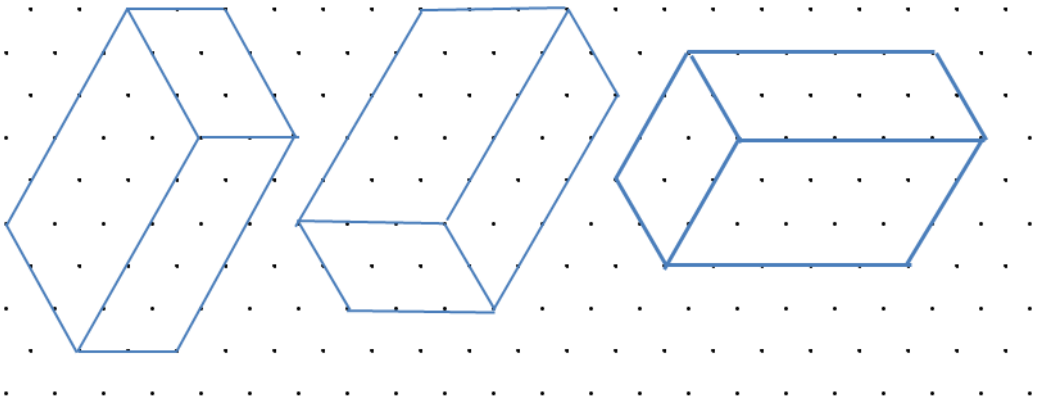






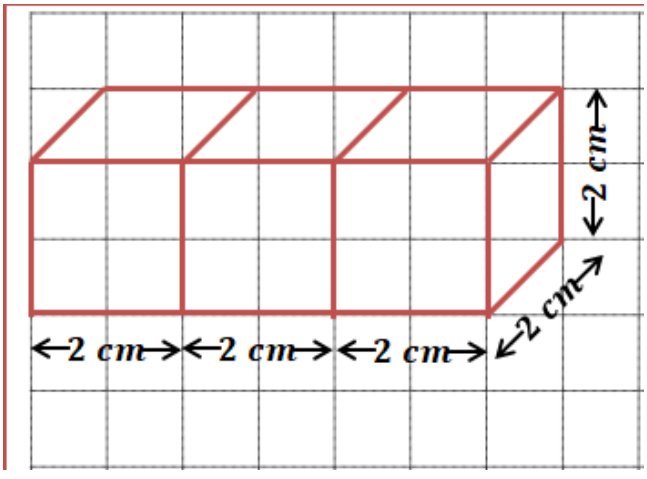
2. The dimensions of a cuboid are 5 cm, 3 cm and 2 cm. Draw three different isometric sketches of this cuboid.

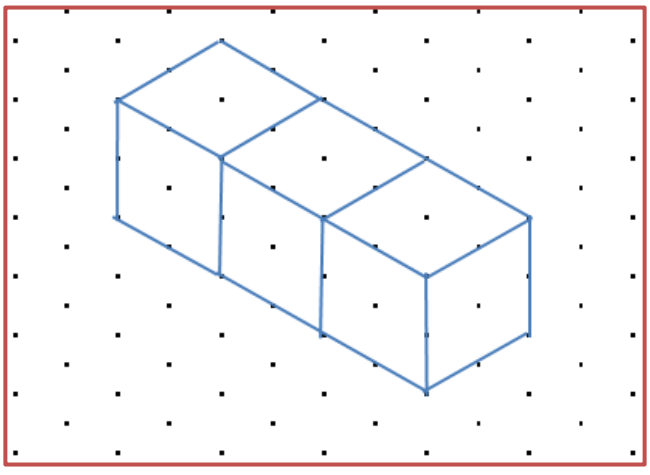
Sol:



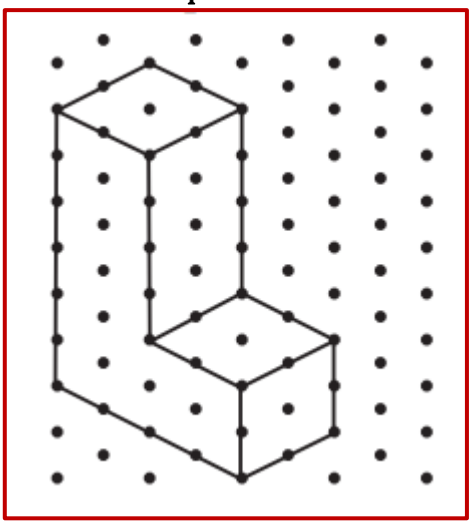
3. Three cubes each with 2 cm edge are placed side by side to form a cuboid. Sketch an oblique or isometric sketch of this cuboid.

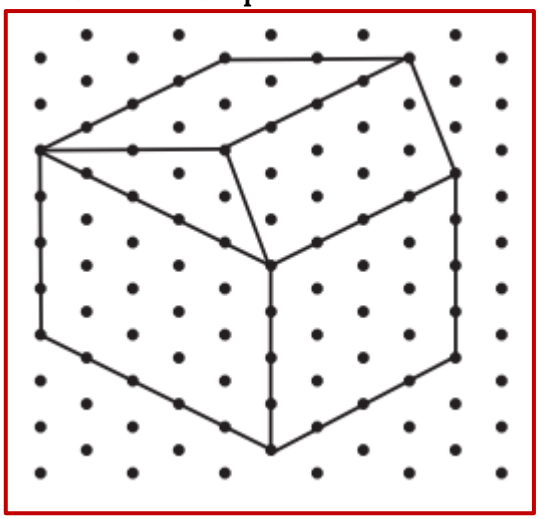
Sol:

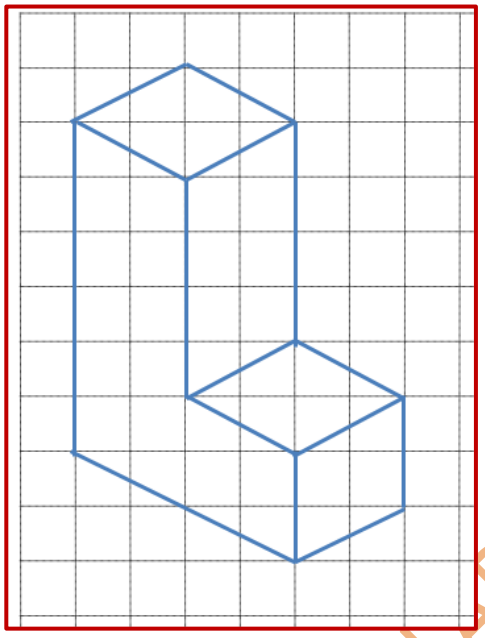


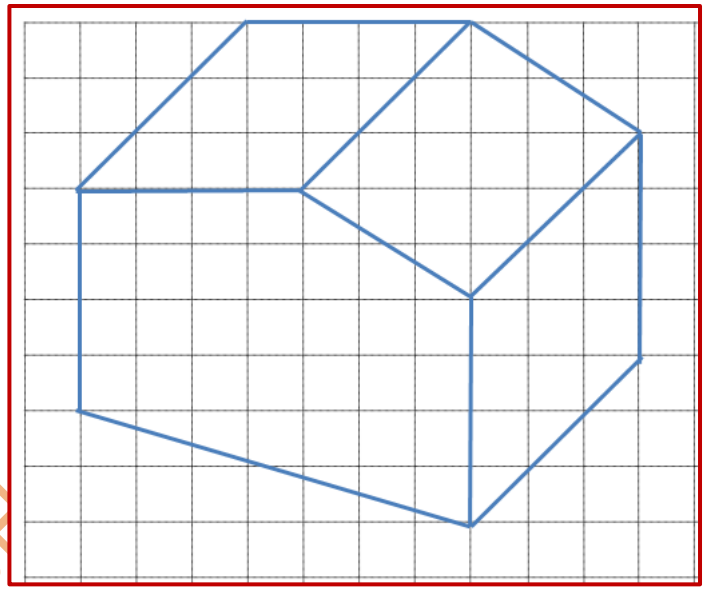


4. Make an oblique sketch for each one of the given isometric shapes:



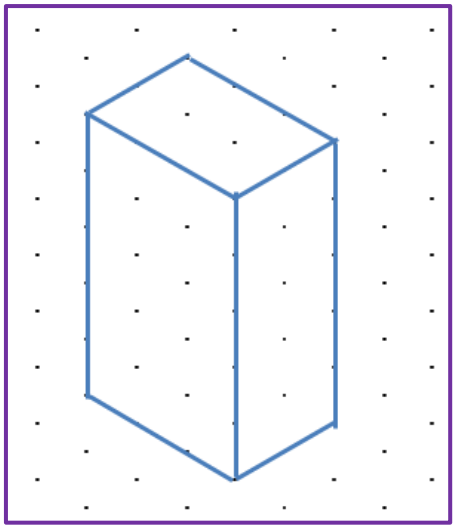


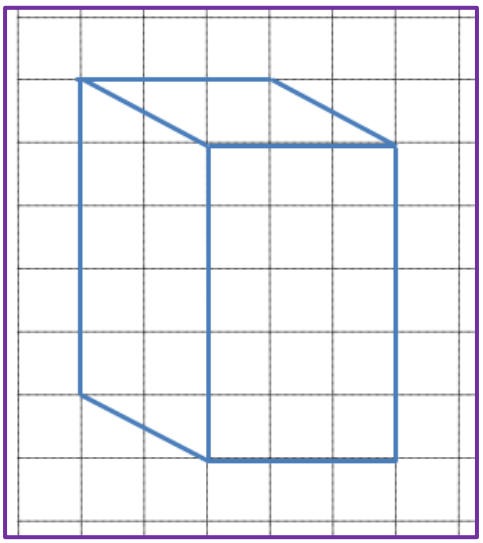




5. Give (i) an oblique sketch and (ii) an isometric sketch for each of the following:
(a) A cuboid of dimensions 5 cm, 3 cm and 2 cm. (Is your sketch unique?) (b) A cube with an edge 4 cm long.

Sol:

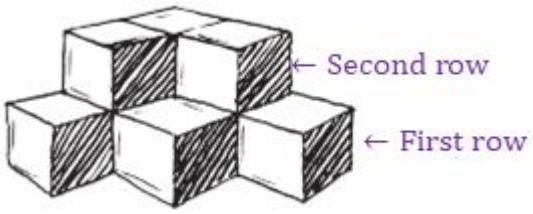




Visualising Solid Objects

Do This

1. **How many cubes?**

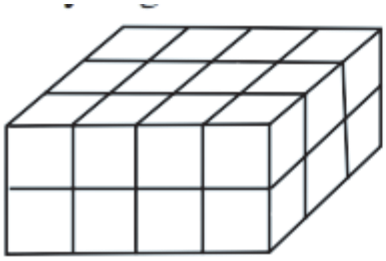


Sol: Number of cubes in first row=6

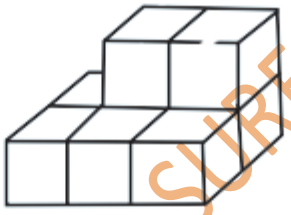
Number of cubes in second row=3

Totalnumber of cubes=6+3=9

2. **Try to guess the number of cubes in the following arrangements**



(i)



(ii)

(iii)

Sol:

(i) Number of cubes in first row=12

Number of cubes in second row=12

Totalnumber of cubes=12+12=24

(ii) Number of cubes in first row=6

Number of cubes in second row=2

Totalnumber of cubes=6+2=8

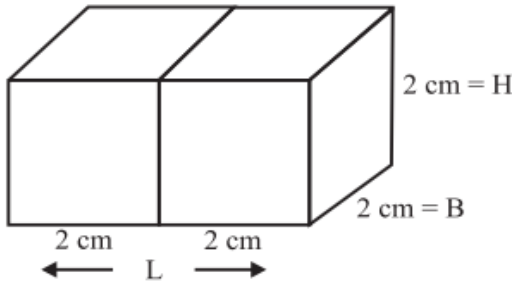
(iii) Number of cubes in first row=8

Number of cubes in second row=2

Totalnumber of cubes=8+2=10

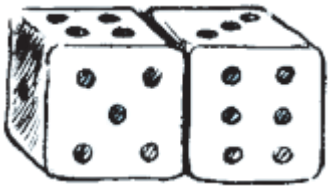
Example 2 : If two cubes of dimensions 2 cm by 2cm by 2cm are placed side by side, what would the dimensions of the resulting cuboid be?

Sol: Length(L)=2+2=4 cm, breadth(B)=2 cm and height(H)=2 cm



TRY THESE

1. **Two dice are placed side by side as shown: Can you say what the total would be on the face opposite to**



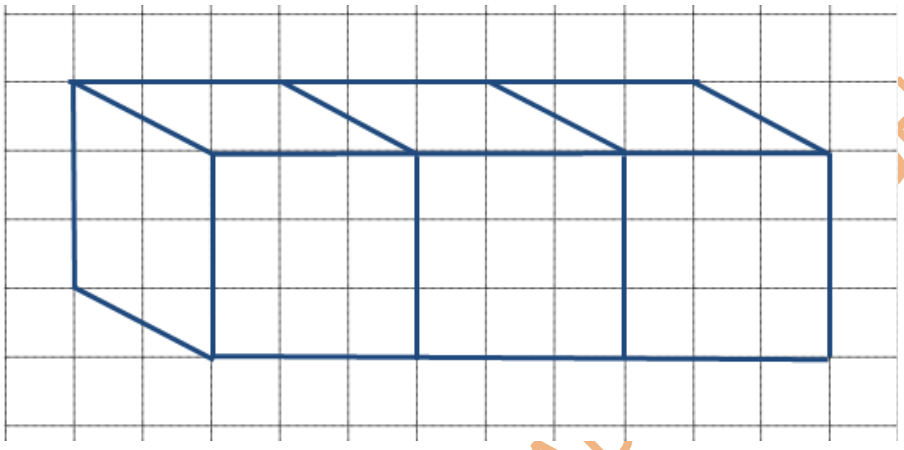
(a) 5 + 6 (b) 4 + 3

Sol: We know that in a dice sum of numbers on opposite faces is 7.

So, the total would be on the face opposite to is (b) 4+3

2. **Three cubes each with 2 cm edge are placed side by side to form a cuboid. Try to make an oblique sketch and say what could be its length, breadth and height.**

Sol:

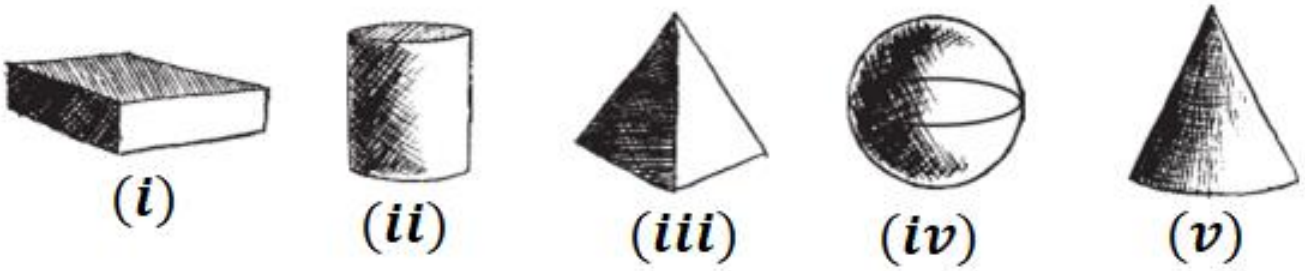


Sol: Length (L)=2+2+2=6 cm, breadth (B)=2 cm and height(H)=2 cm

VIEWING DIFFERENT SECTIONS OF A SOLID

Play this

Make clay (or plasticine) models of the following solids and make vertical or horizontal cuts. Draw rough sketches of the cross-sections you obtain. Name them wherever you can.

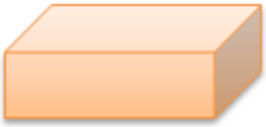
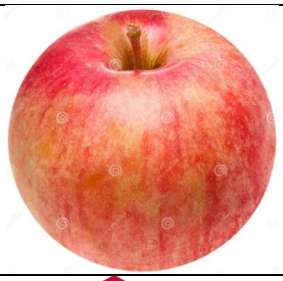
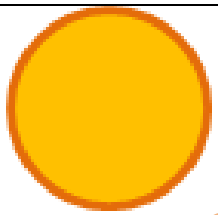
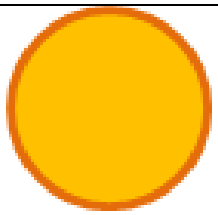
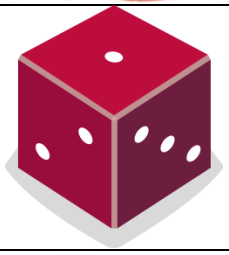
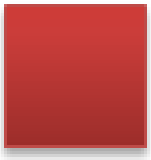
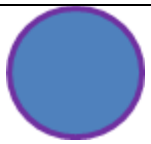


Sol: (i) Rectangle (ii) Circle (iii) Triangle (iv) Circle (v) Circle

EXERCISE 13.3

1. **What cross-sections do you get when you give a**
   (i) vertical cut (ii) horizontal cut to the following solids? (a) A brick (b) A round apple (c) A die (d) A circular pipe (e) An ice cream cone

Sol:

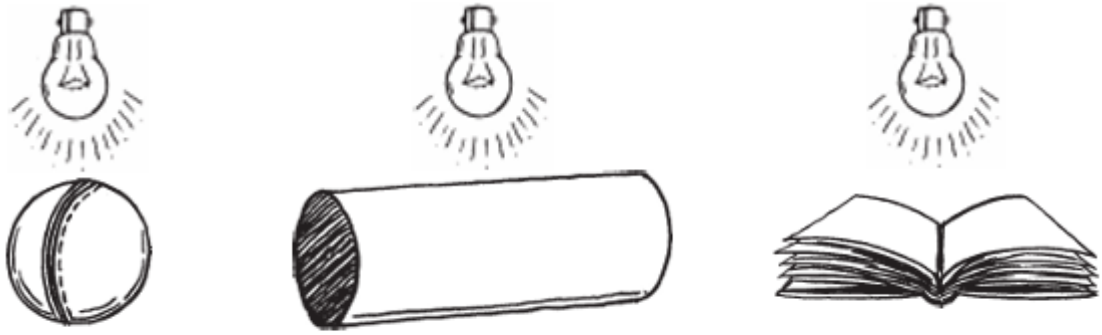
Solid name	Figure	vertical cut	horizontal cut
(a) A brick			
(b) A round apple			
(c) A die			
(d) A circular pipe			
(e) An ice cream cone			

A shadow play

As the light rays hit the 3D object, they create a shadow on a surface, which can be a screen or a plane. The shadow represents a 2D projection of the 3D object.

EXERCISE 13.4

1. **A bulb is kept burning just right above the following solids. Name the shape of the shadows obtained in each case. Attempt to give a rough sketch of the shadow. (You may try to experiment first and then answer these questions).**

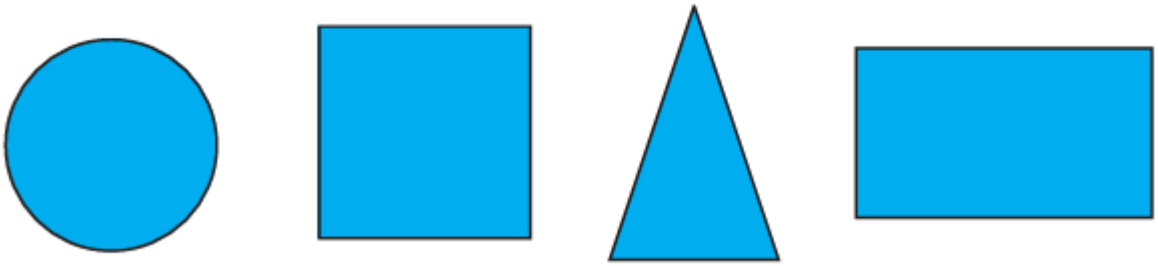


A ball (i) A cylindrical pipe (ii) A book (iii)

Sol: (i) Circle (ii) Rectangle (iii) Rectangle.

2. Here are the shadows of some 3-D objects, when seen under the lamp of an overhead projector. Identify the solid(s) that match each shadow. (There may be multiple answers for these!)

A circle A square A triangle A rectangle



(i) (ii) (iii) (iv)

Sol: (i) Sphere or cylinder vertically (ii) Cube (iii) Cone (iv) Cuboid or cylinder horizontally

3. Examine if the following are true statements:

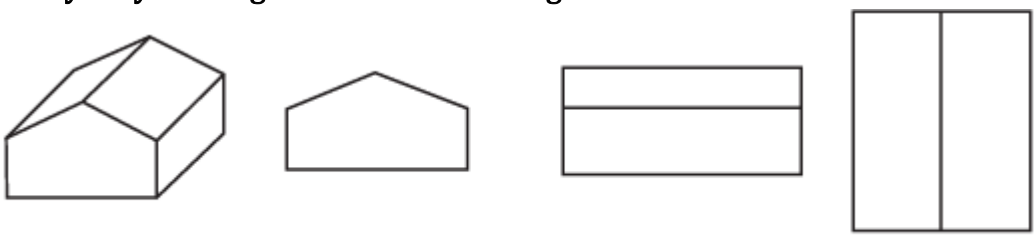
(i) The cube can cast a shadow in the shape of a rectangle.

Sol: The statement is true, when the light is coming diagonally.

(ii) The cube can cast a shadow in the shape of a hexagon.

Sol: False.

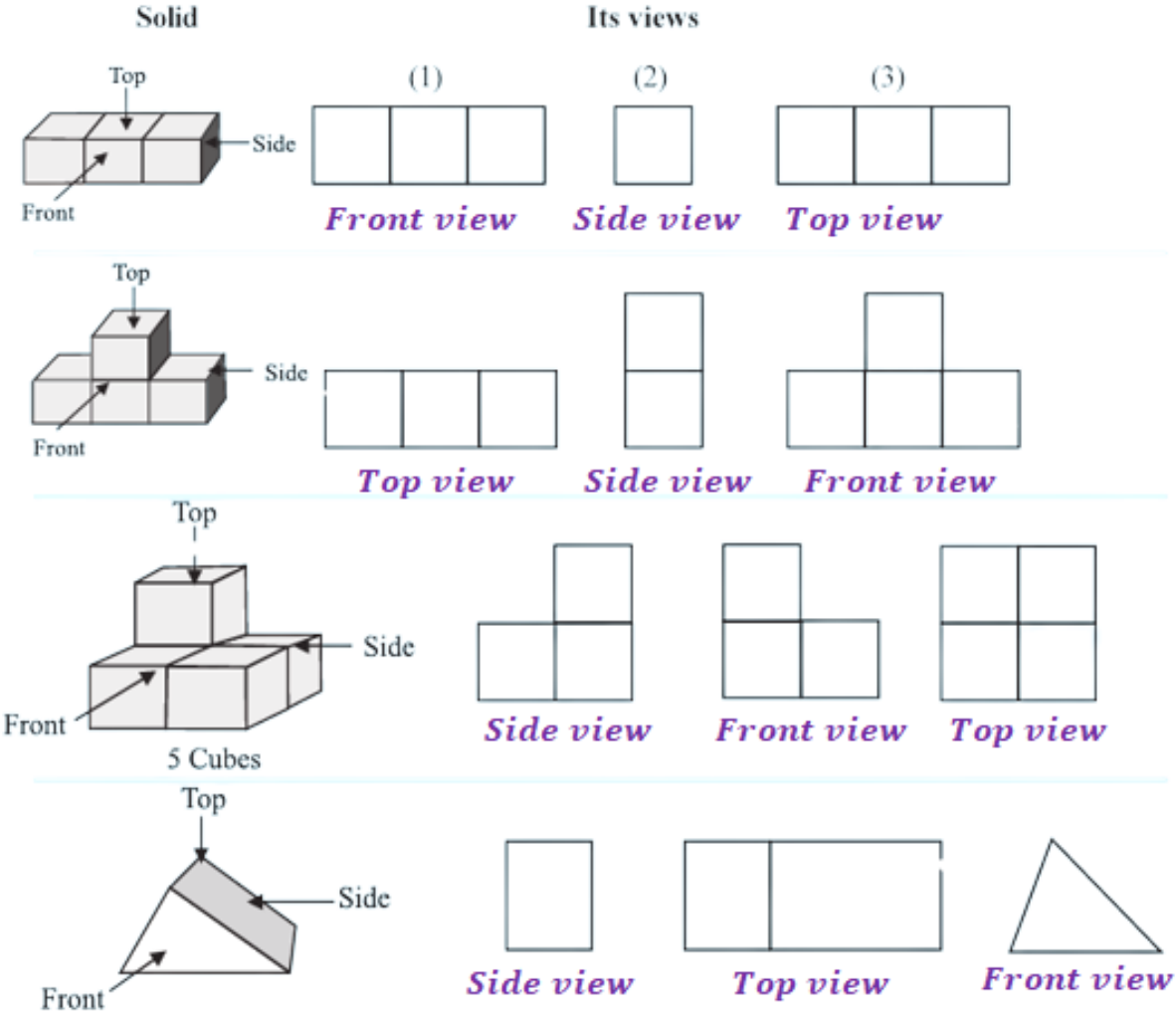
A Third Way is by Looking at it from Certain Angles to Get Different Views



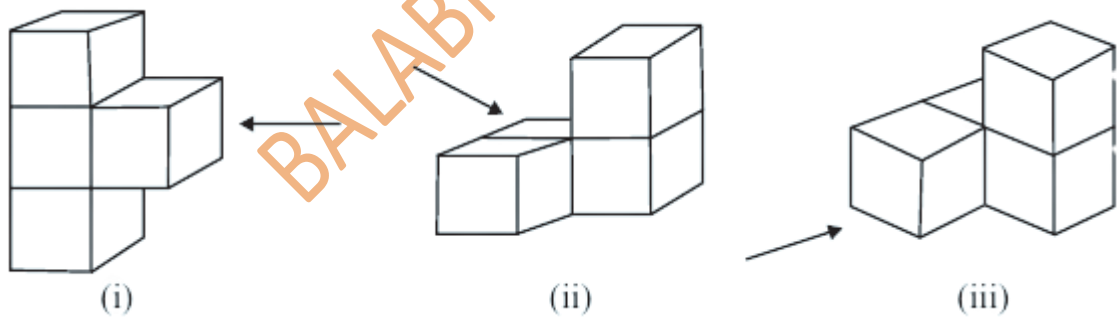
Building Front view Side view Top view

TRY THESE

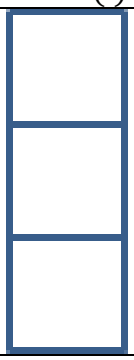
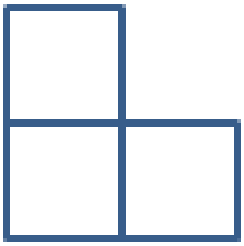
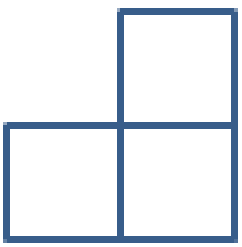
1. For each solid, the three views (1), (2), (3) are given. Identify for each solid the corresponding top, front and side views.



2. Draw a view of each solid as seen from the direction indicated by the arrow.



Sol:

(i)	(ii)	(iii)
		

CHAPTER 14

VII-MATHEMATICS-NCERT-2023-24

14. Brain-Teasers

REPARED BY: BALABHADRA SURESH
https://sureshmathsmaterial.com

1. Solve the number riddles: (i) Tell me who I am! Who I am! Take away from me the number eight, Divide further by a dozen to come up with A full team for a game of cricket!

Sol: $\frac{x-8}{12} = 11 \Rightarrow x - 8 = 11 \times 12 \Rightarrow x = 132 + 8 = 140$

(ii) Add four to six times a number, To get exactly sixty four! Perfect credit is yours to ask for If you instantly tell the score!

Sol: $6x + 4 = 64 \Rightarrow 6x = 60 \Rightarrow x = 10$

2. Solve the teasers: (i) There was in the forest an old Peepal tree The grand tree had branches ten and three On each branch there lived birds fourteen Sparrows brown, crows black and parrots green! Twice as many as the parrots were the crows And twice as many as the crows were the sparrows! We wonder how many birds of each kind Aren't you going to help us find?

Sol: Let parrots $= x$, Crows $= 2x$, Sparrows $= 2 \times 2x = 4x$

Total Number of birds=13×14=182

$x + 2x + 4x = 182 \Rightarrow 7x = 182 \Rightarrow x = \frac{182}{7} = 26$

Parrots $= 26$, Crows $= 2 \times 26 = 52$ and Sparrows $= 4 \times 26 = 104$

(ii) I have some five-rupee coins and some two-rupee coins. The number of two-rupee coins is twice the number of five-rupee coins. The total money I have is 108 rupees. So how many five-rupee coins do I have? And how many two-rupee coins?

Sol: Number of ₹5 coins=x

Number of ₹2 coins=2x

Total amount=108

$(5 \times x) + (2 \times 2x) = 108$

$5x + 4x = 108$

$9x = 108$

$x = \frac{108}{9} = 12$

Number of ₹5 coins=12 , Number of ₹2 coins=2×12=24

3. I have 2 vats each containing 2 mats. 2 cats sat on each of the mats. Each cat wore 2 funny old hats. On each hat lay 2 thin rats. On each rat perched 2 black bats. How many things are in my vats?

Sol: Number of mats in two vats=2×2=4

Number of cats in 4 mats=4×2=8

Number hats with 8 cats=8×2=16

Number of thin rats on 16 hats=16×2=32

Number of black bats on 32 rats=32×2=64

Total number of things in my vats=4+8+16+32+64=124

4. Twenty-seven small cubes are glued together to make a big cube. The exterior of the big cube is painted yellow in colour. How many among each of the 27 small cubes would have been painted yellow on (i) only one of its faces? (ii) two of its faces? (iii) three of its faces?

Sol: If $\boldsymbol{n \times n \times n (= 27 = 3 \times 3 \times 3 \Rightarrow n = 3)}$ cubes are glued together to make a big cube and the big cube is painted a colour then

(a)Number of cubes not painted $= (n-2)^3$

(i)Number of cubes painted only one of its faces $= 6(n-2)^3 = 6(3-2)^3 = 6 \times 1 = 6$

(ii)Number of cubes painted two of its faces $= 12(n-2)^3 = 12(3-2)^3 = 12 \times 1 = 12$

(iii)Number of cubes painted three of its faces $= 8$

5. Rahul wanted to find the height of a tree in his garden. He checked the ratio of his height to his shadow's length. It was 4:1. He then measured the shadow of the tree. It was 15 feet. So what was the height of the tree?

Height	Length of shadow
4	1
x	15 feet

Sol: $x \times 1 = 4 \times 15$ feet

$x = 60$ feet

6. A woodcutter took 12 minutes to make 3 pieces of a block of wood. How much time would be needed to make 5 such pieces?

Sol: 3 pieces→12 minutes

1 piece→ 4 minutes

5 pieces→5×4=20 minutes

7. A cloth shrinks 0.5% when washed. What fraction is this?

Sol: $0.5\% = \frac{0.5}{100} = \frac{0.5 \times 10}{100 \times 10} = \frac{5}{1000} = \frac{1}{200}$

8. Smita's mother is 34 years old. Two years from now mother's age will be 4 times Smita's present age. What is Smita's present age?

Sol: $4 \times$ (Smita's present age $+ 2) = 34 + 2$
$= 36$

Smita's present age $+ 2 = \frac{36}{4} = 9$ years

Smita's present age $= 9 - 2 = 7$ years

9. Maya, Madhura and Mohsina are friends studying in the same class. In a class test in geography, Maya got 16 out of 25. Madhura got 20. Their average score was 19. How much did Mohsina score?

Sol: Mohsina score $= x$

$\frac{16 + 20 + x}{3} = 19$

$36 + x = 57$

$x = 57 - 36 = 21$